

Elastic and Inelastic Behaviour of Materials

Prepared By

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Abstract

Material properties are very important in engineering, as general materials are divided into two groups according to their response to the stress and corresponding strain, brittle materials and ductile material. Materials that show little or no yielding before fracture are referred to as brittle materials, like glass, concrete, and rock, at the beginning when the load applied to this kind of materials the strain will be proportional to applied stress, and when the stress increases more and reaches the yield point after that fracture will happen. Brittle materials show a much higher resistance to the axial compression than their behavior in tension, concrete classified as a brittle material, and its capacity for tensile strength is very low, the characteristics of the stress-strain diagram are mainly related to the mix proportion, time, temperature and curing. While, ductile materials, are any materials that can stand to a large strain before fracture is called ductile materials, like steel, and most metals, in the engineering field ductile materials are desirable for using in the design of the structures, because the ductile material has a capacity to absorb shock or energy and even if they become overloaded they will show a large strain before failure. And this behavior of ductile materials is very useful to preventing the sudden failure in the structures. Dealing with these properties of materials it is very important in the design of structural and the behavior of building. In this study the behavior of materials elastically and plastically is discussed, the stress-strain diagram for different materials ductile and brittle materials is shown and illustrated all the steps and points. The comparison between the elastic and plastic behavior of material is discussed, finally the plastic analysis and plastic design of the rectangular section is determined.

List of Contents

ABSTRACT	I
1- INTRODUCTION	1
1-1 ELASTICITY	1
1-2 PLASTICITY	6
1-3 ELASTIC DEFORMATION	7
1-4 PLASTIC DEFORMATION	8
1-5 HOOKE'S LAW	10
1-6 MODULUS OF ELASTICITY	12
2-1 CONVENTIONAL STRESS-STRAIN DIAGRAM	13
2-3 DEFINITION OF EACH PART OF STRESS-STRAIN DIAGRAM	16
2-3-1 Proportional limit	17
2-3-2 Elastic Limit	17
2-3-3 Yield Stress Point	17
2-3-4 Ultimate Stress Point	17
2-3-5 Breaking Stress Point (Point of Fracture)	18
3- BEHAVIOR OF MATERIALS	19
3-1 DUCTILE MATERIALS	19
3-2 BRITTLE MATERIAL	21
4- PLASTIC ANALYSIS [6,7]	22
4-1 BACKGROUND	22
4-2 BASIS OF PLASTIC DESIGN	22
4-2-1 Material Behavior	22
4-2-2 Cross Section Behavior	24
4-2-3 Plastic Hinge	26
4-3 ANALYSIS OF RECTANGULAR CROSS SECTION	27
4-3-1 Elastic Moment	28
4-3-2 Elasto-Plastic Moment	28
4-3-3 Plastic Moment	30
CONCLUSIONS	31
REFERENCES	32

List of Figures

FIGURE 1- ELASTIC BEHAVIOR OF MATERIALS.....	1
FIGURE 2-PLASTIC BEHAVIOR OF MATERIALS	6
FIGURE 3- STRESS STRAIN DIAGRAM	7
FIGURE 4-TENSILE TEST	8
FIGURE 5- ELASTIC LIMIT	8
FIGURE 6- HOOKS LAW, WHERE THE APPLIED FORCE F EQUALS CONSTANT K TIME THE DISPLACEMENT OR CHANGE IN LENGTH X.....	10
FIGURE 7- HOOK'S LAW FOR ELASTIC SOLIDS	11
FIGURE 8- CONVENTIONAL AND TRUE STRESS-STRAIN DIAGRAM FOR DUCTILE MATERIALS (STEEL) (NOT TO SCALE)	14
FIGURE 9- TYPICAL TENSILE-TEST SPECIMEN	14
FIGURE 10- TEST SPECIMEN WITH TENSILE LOAD	15
FIGURE 11- THIS MACHINE IS USED TO TEST TENSILE-TEST SPECIMENS	15
FIGURE 12- STRESS-STRAIN DIAGRAM	16
FIGURE 13- STRESS-STRAIN DIAGRAM FOR BRITTLE AND DUCTILE MATERIALS.....	18
FIGURE 14- NECKING OF A MILD-STEEL BAR IN TENSION	18
FIGURE 15- STRESS-STRAIN DIAGRAM OF TWO TYPICAL DUCTILE MATERIALS.....	19
FIGURE 16-TEST SPECIMEN OF A DUCTILE MATERIAL	20
FIGURE 17- STRESS-STRAIN DIAGRAM FOR A TYPICAL BRITTLE MATERIALS	21
PHOTO 18-TESTED SPECIMEN OF A BRITTLE MATERIAL	21
FIGURE 19- STRESS-STRAIN DIAGRAMS FOR DUCTILE MATERIAL (STEEL).....	22
FIGURE 20- IDEALIZED STRESS-STRAIN DIAGRAM.....	23
FIGURE 21- CROSS SECTION STRESSES AND MOMENT-ROTATION CURVE.....	24
FIGURE 22-IDEALIZED STRESS-STRAIN DIAGRAM	26
FIGURE 23-STRESS DISTRIBUTION OF A RECTANGULAR SECTION IN ELASTIC, ELASTO-PLASTIC, AND PLASTIC	27

1- Introduction

1-1 Elasticity

Elasticity is the ability of a solid material to go back to its original form after the forces that caused the deformation is removed. A material with this property is said to behave elastically. Most of the solid materials show elastic behavior, but there is a limit to the magnitude of the force and the accompanying distortion which is elastic return is applicable to any given material. This limit called the elastic limit, in this limit, the material carries the maximum stress or force per unit area within a solid material before going to the plastic limit and start of permanent deformation. Stresses after the elastic limit cause a material to yield. For such materials, the elastic limit shows the end of elastic behavior and the starting of plastic behavior. After the elastic limit in the brittle material when more Stresses were applied cause a fracture with almost no plastic deformation. The elastic limit depends significantly on the type of material considered; for example, a steel bar or wire can be extended elastically only about 1% of its original length, while for a piece of certain rubberlike materials, elastic extensions of up to 1000% can be achieved [1].

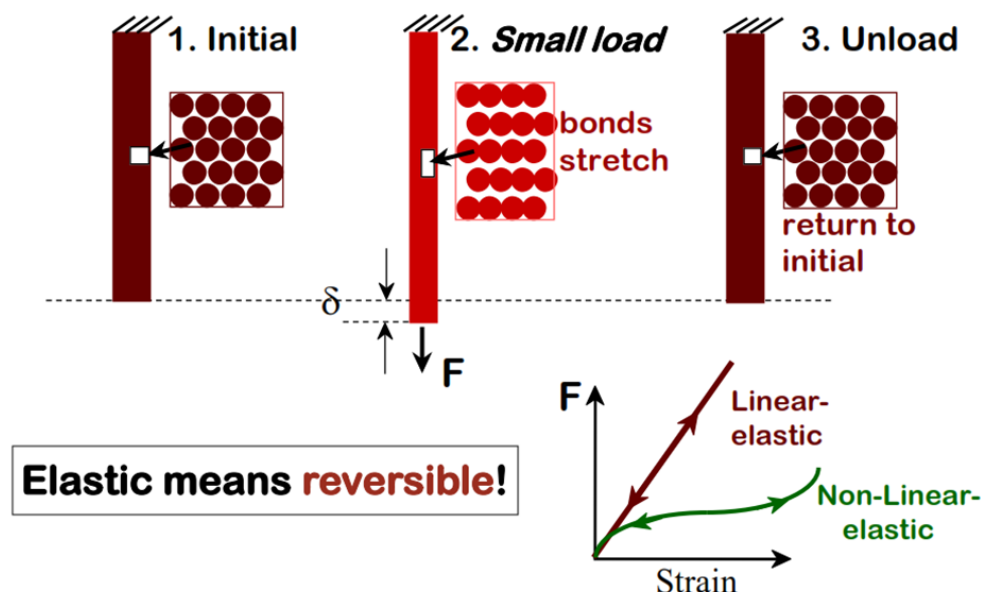


Figure 1- Elastic Behavior of Materials

1-2 Plasticity

Is that the ability of materials to flow or to change form permanently when exposed to stresses of an intermediate value between those generating short-term deformation, or elastic behavior, and those causing failure of the material, or rupture. Plasticity permits solid materials under the action of external forces to undergo permanent deformation without rupture. Elasticity, in comparison, is the ability of solid materials to return to its original form after the load that cause the distortion is removed. Plastic distortion happens in some metal-forming processes such as (rolling, pressing, and forging) and in geological processes (rock folding and rock flow within the earth under very high pressures and at elevated temperatures). Plastic deformation is a behavior of ductile and malleable solids. Brittle materials, like cast iron, can't be permanently deform, however at elevated temperatures some, such as glass, which is not a crystallized solid, can undergo plastic flow [1].

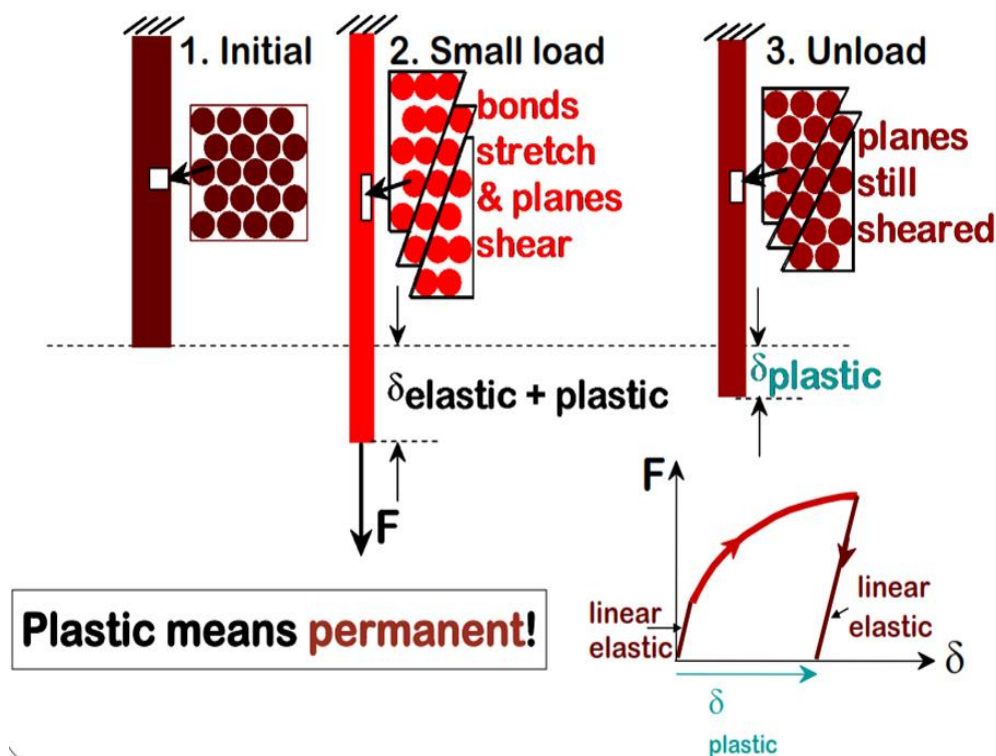


Figure 2-Plastic Behavior of Materials

1-3 Elastic Deformation

Elastic deformation is the deformation that disappears when the external forces that caused the change and the stress are removed. Elastic distortion is hence reversible and non-permanent. Elastic distortion is best clarified by the chemical idea “elasticity”. Elasticity is the ability of a material to return to its original state after deformation. Elastic deformation depends mainly on the chemical bonding of the materials. If the chemical bonds are able to withstand a high stress by deforming without fracture, that material can undergo elastic distortion. The figure below shows the stress-strain curve for a ductile material. Such as copper metal. The elastic region shows the extent to which the elastic distortion takes place. After the elastic limit, the material will undergo plastic distortion which is permanent [2,8].

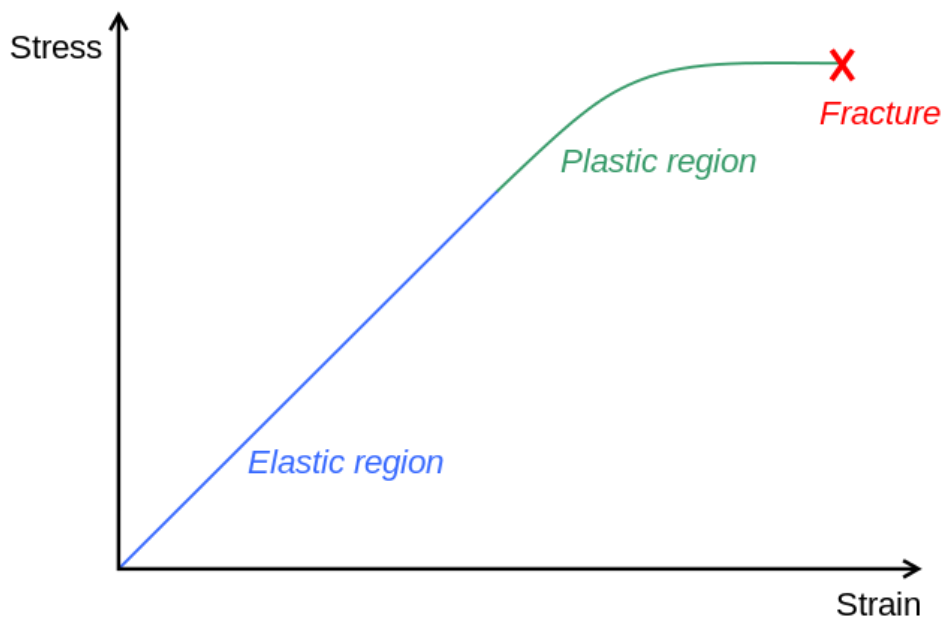


Figure 3- Stress Strain Diagram

1-4 Plastic Deformation

Plastic deformation is the permanent distortion or change in the form of a solid body without fracture under the action of a continued load. This situation happens when the material is subjected to a very large stress. Plastic deformation is best explained by the chemical concept “plasticity”. Plasticity is the quality of a material that being easily shaped or molded permanently. Plastic distortion happens because of the rupture of a limited number of chemical bonds between atoms that composite the materials. Because of plastic distortion atoms of the materials are sliding on each other. This causes disarrangement of atoms; therefore, the material stays in its situation after removing the applied stress. For ductile materials, end of the elastic limit is the starting point of plastic distortion. If the stress is applied more than the elastic limit, then the permanent distortion in the material will occur. Metals, plastics, and rocks are materials that under the load permanent deformation occurs and it can be observed clearly. In ductile materials such as metals permanent change happens when the load that makes a deformation exceeds the elastic limit. But in brittle materials such as rocks, no elastic distortion can be seen before the starting permanent distortion. The elastic limit is shown in the Fig.(5) below [2,8].

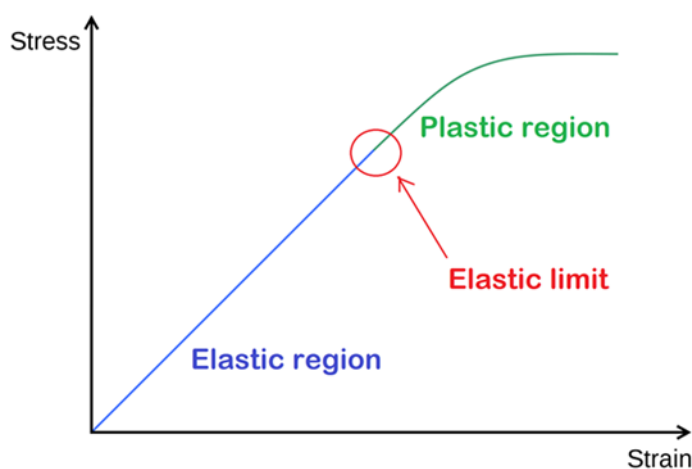


Figure 5- Elastic Limit

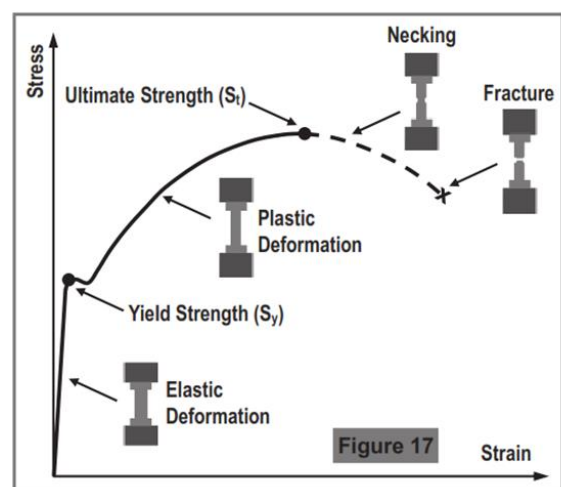


Figure 4-Tensile Test

Comparison of Plastic and Elastic [2]

	Plastic	Elastic
Definition	Plastic deformation is the permanent distortion or change in form of a solid body without fracture under the action of a continue force	Elastic deformation is the change in the form of materials that disappears when the load that causes the distortion is removed.
Process	After deformation did not return to it is original shape.	Return to it is original shape.
Ductility	They are highly ductile in nature.	It is less ductile in nature.
Resilience	They have little yield strength.	They have high yield strength.
Modulus of elasticity	The ratio of stress to strain is large.	The ratio of stress to strain is less or same.
Toughness	They do not capable to absorb energy up to a fracture.	They have the capability to absorb energy up to a fracture.
Bonds	The molecular bonds are fractured. Cause some of the chemical bonds of the substance to undergo breakage	The molecular bonds do not get fractured. Causes the chemical bonds of the substance to undergo stretching and bending
Atoms	Atoms slide on each other	Atoms do not slide on each other
Shape and size	The shape and size changes permanently.	The shape and size does not change permanently,
Stress Relation	While plastic deformation hold a curved relationship having a peak	Elastic deformation holds a linear relationship with stress
Example	Plastics	Rubber.

1-5 Hooke's law

the law of elasticity exposed by an English scientist Robert Hooke in 1660, which states that, for comparatively small deformations of a body, the displacement is directly relative to the deforming force or load. According to this conditions, the materials returns to its original form when the force is removed. Elastic behavior of solids according to Hooke's law can be defined by the fact that a small change of their atoms or ions from original positions is also proportional to the load that causes the distortion or change.

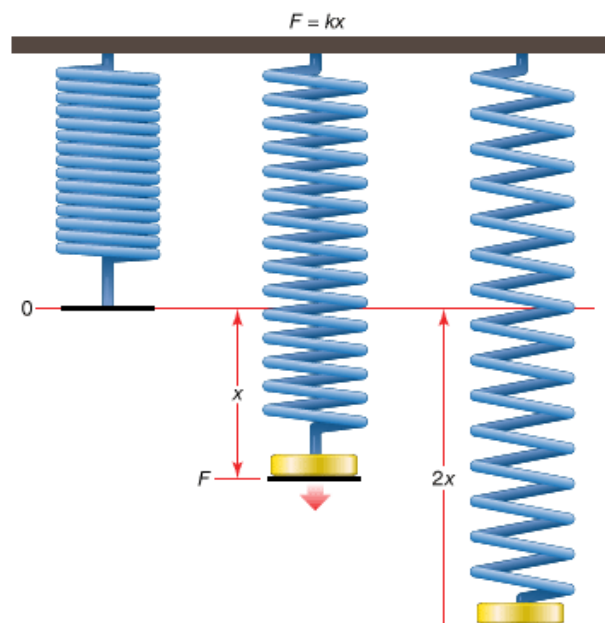
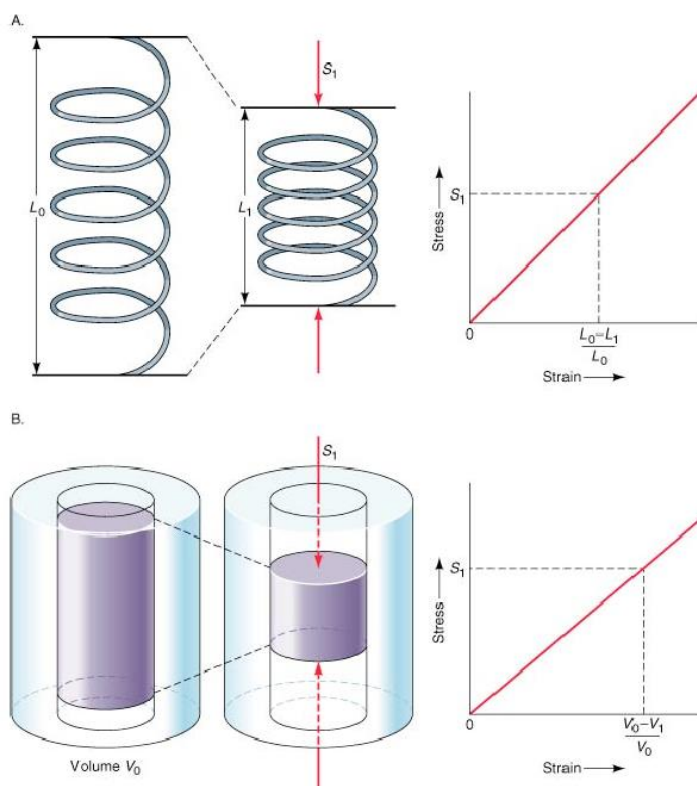


Figure 6- Hooke's Law, where the applied force F equals constant k times the displacement or change in length x .

The distortion loads are applied to a substance in away cause stretching, compressing, squeezing, bending, or twisting to the material. Therefore, a metal wire shows elastic behavior according to Hooke's law because of the small increase in its length when stretched by an applied force. Scientifically, Hooke's law states that the applied force F equals a constant k times the displacement or change in length X , or $F = k x$. The value of a constant k affected by the type of elastic material under consideration and also on its size and shape. The deformation of the elastic material is frequently more than that expected on the basis of Hooke's law, when a large amount of the load is applied to the material,

even though the material stay in elastic limit and return to its original form after the load is remove. The elastic properties of solids illustrate by Hooke's law only in the limit in which the stress and displacement are proportional to each other.

Hooke's law also can be expressed in terms of stress and strain. Stress is a force per unit area that develops due to an external force. The strain is the relative deformation developed by stress. In the elastic limit for relatively small load, the value of stress is proportional to the magnitude of strain [1,2,4,8].



Hooke's Law for elastic solids

Figure 7- Hooke's Law for Elastic Solids

1-6 Modulus of elasticity

One of the very important properties of solid material is the modulus of elasticity (Young's modulus) E is a material property that describes its stiffness. In most of engineering work the buildings and body are allow to relatively small deformations, involving only the elastic portion of the corresponding stress-strain diagram which is stress is comparative to strain. Mechanical deformation puts power into a substance. The energy is kept elastically or dissipated plastically. The way material stores this power is summarized in stress-strain curves. Stress is a force divided by unit area and strain define as elongation or contraction per unit length when a material deforms elastically, the amount of distortion also related to the size and shape of the substance, but the strain for a given load is continuously the same and the two are related by Hooke's Law (stress is directly relative to strain). The largest value of the stress for which Hooke's law can be used for a given substance is defined as the elastic limit of that material.

$$\sigma = E \cdot \varepsilon$$

Where:

σ is stress [MPa]

E modulus of elasticity [MPa]

ε strain [unit less]

From the Hook's law, the modulus of elasticity is defined as the ratio of the stress divided by strain

$$E = \frac{\sigma}{\varepsilon}$$

The relation is defined as Hooke's law, after Robert Hooke (1635–1703), an English scientist the coefficient E is called the modulus of elasticity of the material, or also Young's modulus, after the English scientist Thomas Young (1773–1829). Since the strain ε is a dimensionless quantity, the modulus E is expressed in the same units as the stress σ [2,3,9].

2- Stress-Strain Diagram

When we got the stress and strain reading from the test, then the results can be plotted to draw a curve called the **stress–strain diagram**. This diagram is very useful since it applies to a sample of the material made of any size.

2-1 Conventional Stress–Strain Diagram

The **engineering stress** is determined by dividing the applied load **P** by the specimen's original cross-sectional area **A₀**.

$$\sigma = \frac{P}{A^{\circ}}$$

Similarly, the **strain** can be read directly from the strain gage reading, or can be found by dividing the change in the specimen's length δ , by the specimen's original length L_0 . Thus,

$$\epsilon = \frac{\delta}{L^{\circ}}$$

When these values of σ and ϵ are plotted, where the vertical axis is the stress and the horizontal axis is the strain, the obtained curve is called a conventional stress–strain diagram. A typical example of this curve is shown in Fig.8 however; two stress-strain curves for specific material may be similar to each other but never will be the same. This is because the results actually depend upon such variables as the material's composition, microscopic imperfections, the way the specimen is manufactured, the rate of loading, and the temperature during the time of the test.

From the curve in Fig.(8), we can identify four different regions in which the material behaves in a unique way, depending on the amount of strain induced in the material [2,4].

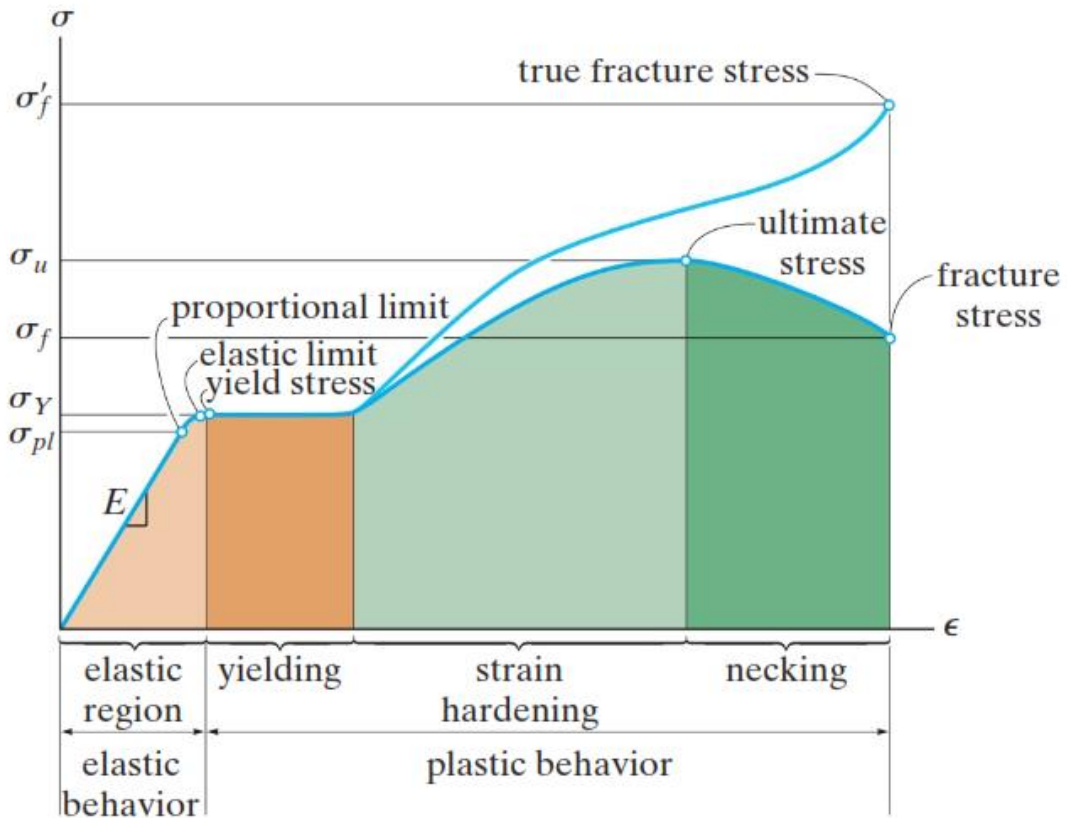


Figure 8- Conventional and true stress-strain diagram for ductile materials (Steel) (Not to scale)

The diagram above showing the stress and strain relation, for a given material stress-strain curve is a very important properties. To determine the stress-strain diagram of a material, frequently performs a tensile test on a sample of the material. One type of specimen commonly used as shown in Fig.(9). The area of the sample cross-section in precisely determined, and two gage marks have been engraved on that portion at a distance L_0 from each other. The distance L_0 is known as the gage length of the specimen.

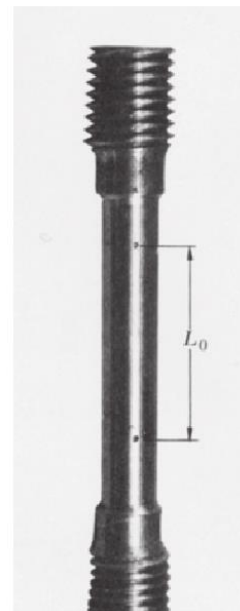


Figure 9- Typical Tensile-test Specimen

The test sample is then put in a testing machine as shown in the Fig.(11), below which is used to apply a centric load P . As the load P increases, the distance L between the two gage marks also increases as shown in the Fig.(10). The distance L is measured with a dial gage, and the elongation $\delta = L - L^{\circ}$ is recorded for each value of P . A second dial gage is often used at the same time to measure and record the change in diameter of the specimen. From each couple of readings P and δ , the stress σ is determined by dividing load P by the original cross-sectional area A_0 of the sample, and the strain is calculated by dividing the elongation δ by the original distance L_0 between the two gage marks. The stress strain diagram may be obtained by plotting strain in X-axis and stress in Y-axis. Each material has different Stress-strain diagrams and widely varies from other materials, and different tensile tests performed on the same material may obtain different stress-strain diagram, the result is depending upon the temperature of the specimen and the speed of loading. From the stress-strain diagram the materials can be divided into two different groups known as brittle materials, such as glass and concrete, the other group ductile materials such steel and aluminum [2,3].



Figure 11- This machine is used to test tensile-test specimens



Figure 10- Test specimen with tensile load

2-3 Definition of Each Part of Stress-Strain Diagram

A stress-strain curve is a property of the material when it is subjected to load. The stress-strain diagram is obtained by plotting stress along Y-axis and the strain along X-axis, as shown below in the Fig.(12)

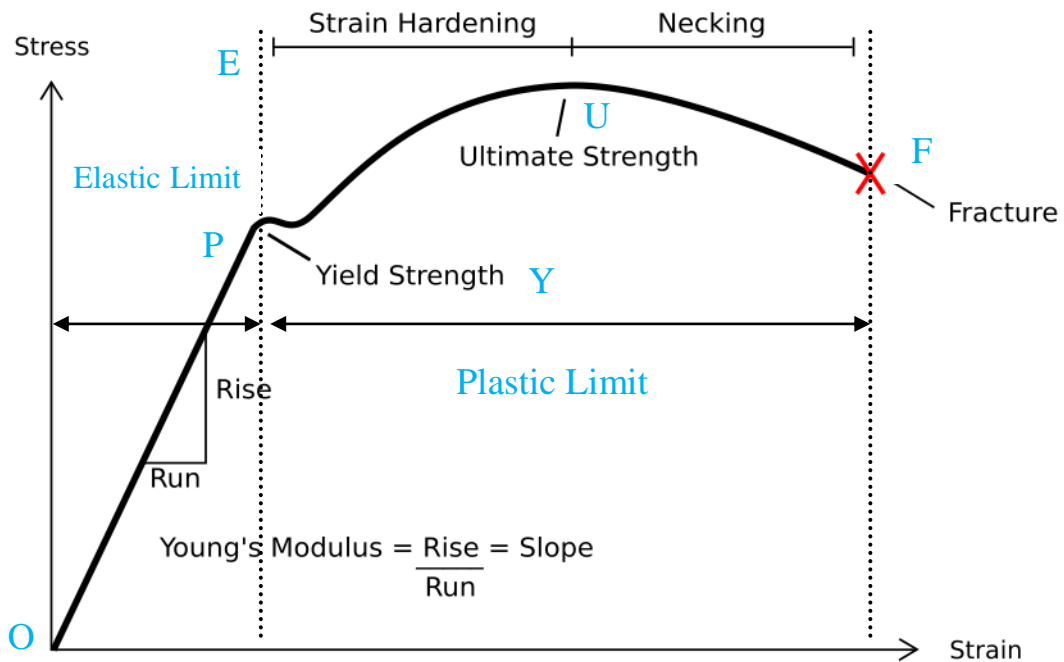


Figure 12- Stress-Strain diagram

From the stress-strain diagram above different points and stage can be seen on the stress-strain curve, when a ductile material such as mild steel is exposed to tensile force under the tensile testing machine, and then it undergo various periods before fracture.

Stages are:

1. Proportional Limit
2. Elastic Limit
3. Yield Point
4. Ultimate Stress
5. Breaking Point
6. Plastic Limit

2-3-1 Proportional limit

Proportional limit is a point on the curve of stress-strain diagram which is the change in stress is relative to strain. From the stress-strain curve, point **P** is the defined as the proportional limit or it can also be identified as the limit of proportionality. The stress till this point also called as proportional limit stress. Hook's law of proportionality from diagram can be defined between point **O** and **P**. It is so because **O** and **P** is an inclined line which shows that Hook's law of stress-strain is followed up to point **P**.

2-3-2 Elastic Limit

Elastic limit is the limit that the material under the stress of a specific value is totally elastic, mean by removing the external load the materials return to it is original shape and size. And as shown in the diagram, that point **E** is the elastic limit point.

2-3-3 Yield Stress Point

Yield stress is defined as the stress after which material extension takes place more quickly with no or little increase in load. Point **Y** is the yield point on the diagram and stress related with this point is known as yield stress.

2-3-4 Ultimate Stress Point

Ultimate stress point is the maximum strength that materials have to stand stress before breaking. On the graph point, **U** is the ultimate stress point. After point **U** materials have very low or zero strength to face more stress.

2-3-5 Breaking Stress Point (Point of Fracture)

Breaking point or breaking stress is a point where the strength of material breaks. The stress related to this point known as breaking strength or rupture strength. On the stress-strain curve, point **F** is the fracture stress point [5].

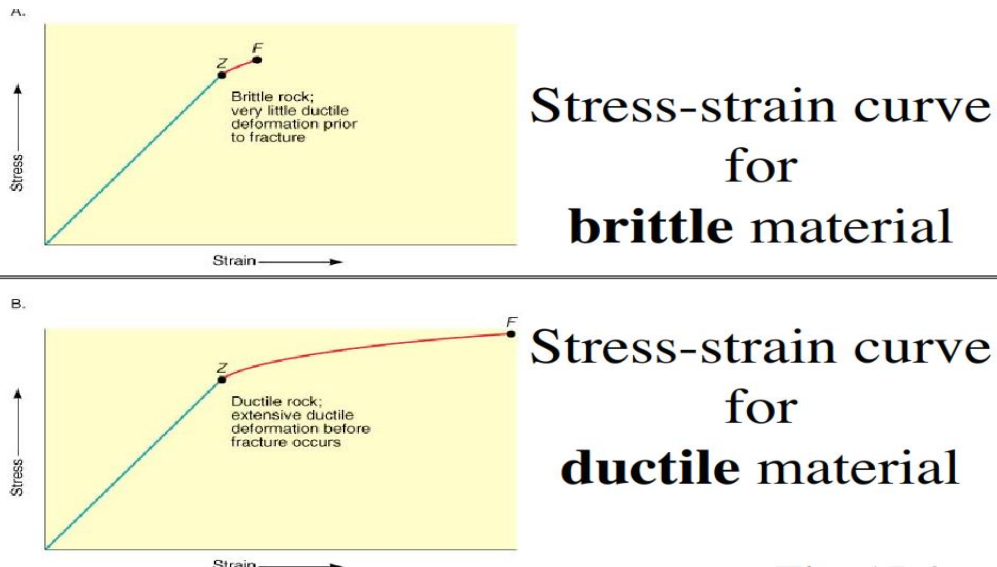


Figure 13- Stress-Strain diagram for brittle and ductile materials

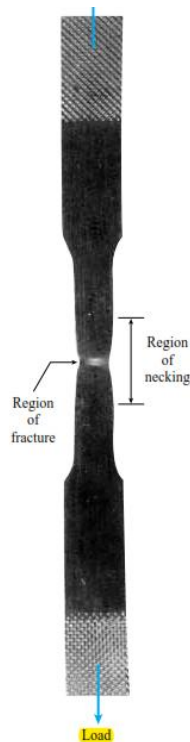


Figure 14- Necking of a mild-steel bar in tension

3- Behavior of Materials

3-1 Ductile Materials

Ductile material includes structural steel, in addition to many strips of other metals, are recognized by their capability to yield at room temperatures as the sample is exposed to an external force, its length starts to increase relatively with the increasing load and at a very slow rate. Therefore, the initial portion of the stress-strain diagram is a straight line with a steep slope as shown in Fig.(15). However, when the stress reaches the yield point σ_Y and after that a small increasing in load cause the specimen to undergoes a large deformation. As can be note from the stress-strain diagrams of two typical ductile materials Fig.(15). The deformation of the sample after reach to the yield point can be shown 200 times larger as its deformation before yield.

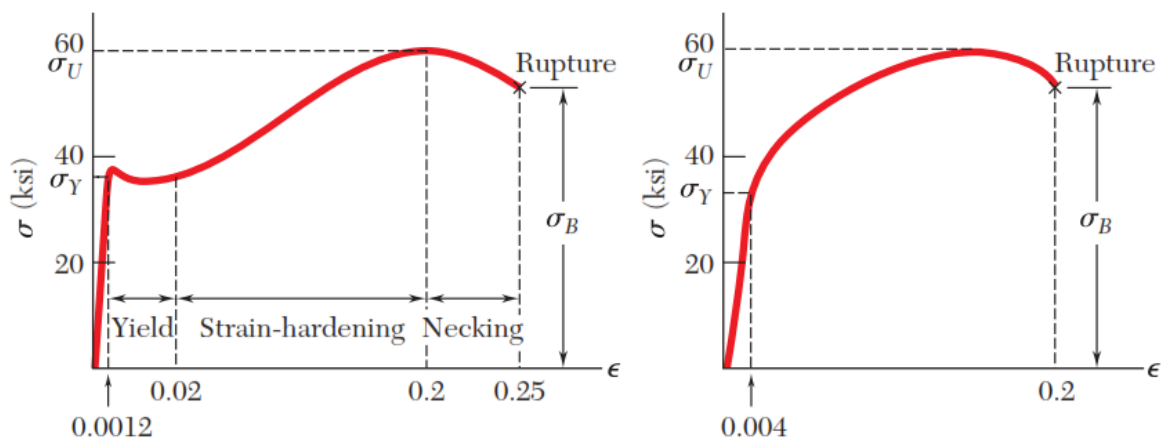


Figure 15- Stress-strain diagram of two typical ductile materials
(a) Low-carbon steel, (b) Aluminum alloy

By increasing the load to the certain value, the cross-section of a portion of the sample starts to decrease, because of local instability as shown in the (Photo.16). This happening is known as necking. When the necking of the material sample is started a small amount of load is enough to keep the sample elongation further until is finally fracture as shown in the Fig.(16) it notes that rupture happens along a cone-shaped surface that forms an angle of approximately 45° with the original surface of the sample. This shows that shear is mainly responsible for the fracture of ductile materials, and confirms the fact that,

under an axial load, shearing stresses are largest on surfaces forming an angle of 45° with the load. The stress at which yield is started is called the yield strength of the material, the stress equivalent to the maximum load applied to the specimen is known as the ultimate strength, and the stress equivalent to fracture is known as the breaking strength [3,9].

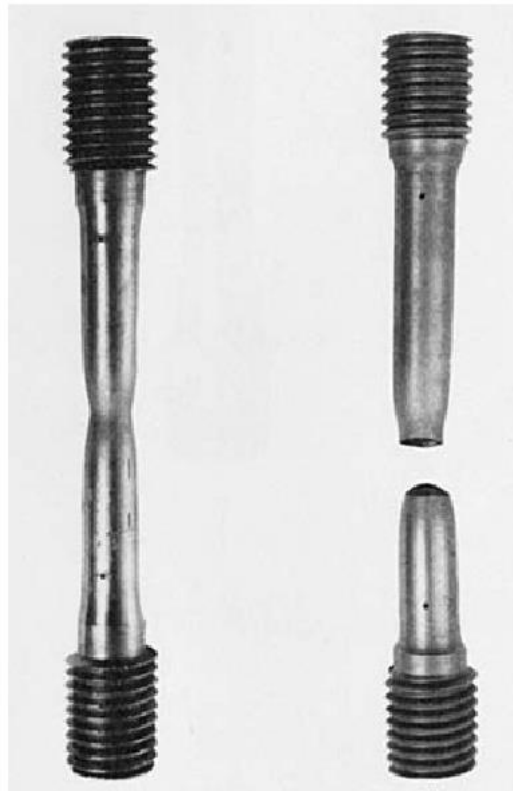


Figure 16-Test specimen of a ductile material

3-2 Brittle Material

Brittle materials, such as cast iron, glass, and stone, are recognized by the behavior that failure happens without any noticeable distortion in the rate of elongation as represented in Fig.(17). From the stress-strain diagram for a brittle material, it can be note that there is no difference in ultimate stress and yield stress because of the brittle material undergoes a little or no distortion before failure. Also, the deformation at the time of failure is much smaller for brittle than for ductile materials. From Fig.(18), we note the lack of any necking of the sample in the case of a brittle material and note that fracture take place along a surface normal to the load. We determine from this observation that normal stresses are mainly responsible for the failure of brittle materials [3,9].

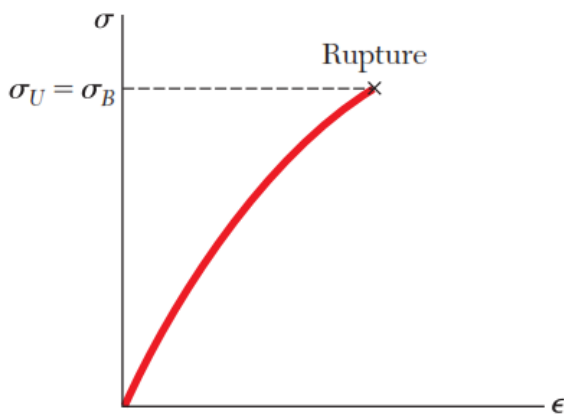


Figure 17- Stress-strain diagram for a typical brittle materials



Photo 18-Tested specimen of a brittle material

4- Plastic Analysis [6,7]

4-1 Background

Until now we have focused on the elastic analysis and design of structures. In these analyses we used superposition often, knowing that for a linearly elastic structure it was effective. However, an elastic analysis does not show enough evidence about the value of the loads that will actually cause a collapse to the structure. A complex structure may carry loads more than the load that first causes a yield to occur at any point in the structure. Actually, a structure will stand as long as it is able to find excess to yield. The plastic analysis is the method that the actual load that cause failure to the structure is calculated, and this load that cause failure is significantly greater than the elastic load capacity. In plastic analysis and design of a structure, the ultimate load of the structure as a total is taken as the design criterion. Plastic analysis and design has its main application in engineering to analysis and design of complex structures.

4-2 Basis of Plastic Design

4-2-1 Material Behavior

A uniaxial tensile stress on a ductile material such as mild steel typically provides the following graph of stress versus strain:

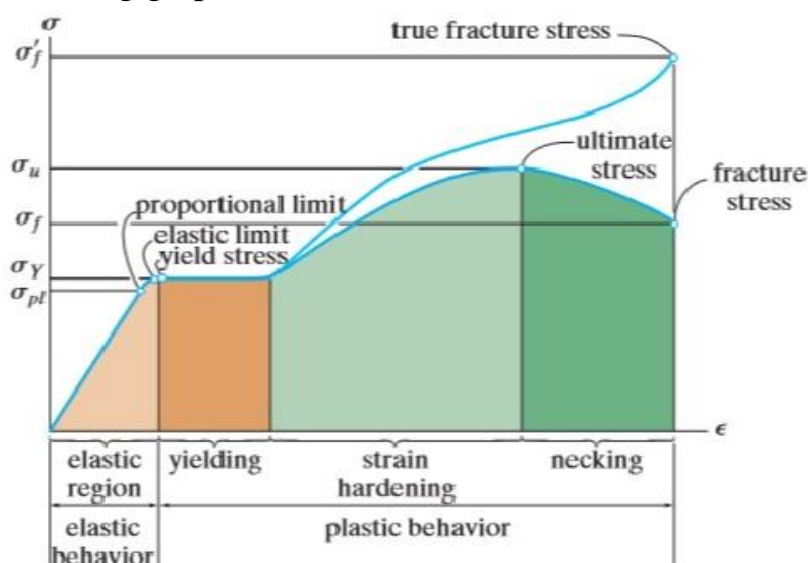


Figure 19- Stress-strain diagrams for ductile material (steel)

As can be observe, the material can bear strains far in excess of the strain at which yield occurs before failure. This property of the material is known as ductility. However complex models do exist to precisely reflect the above real behavior of the material, the most common and simplest model is the idealized stress-strain curve. This is the curve for an ideal elastic-plastic material as represented in the graph below.

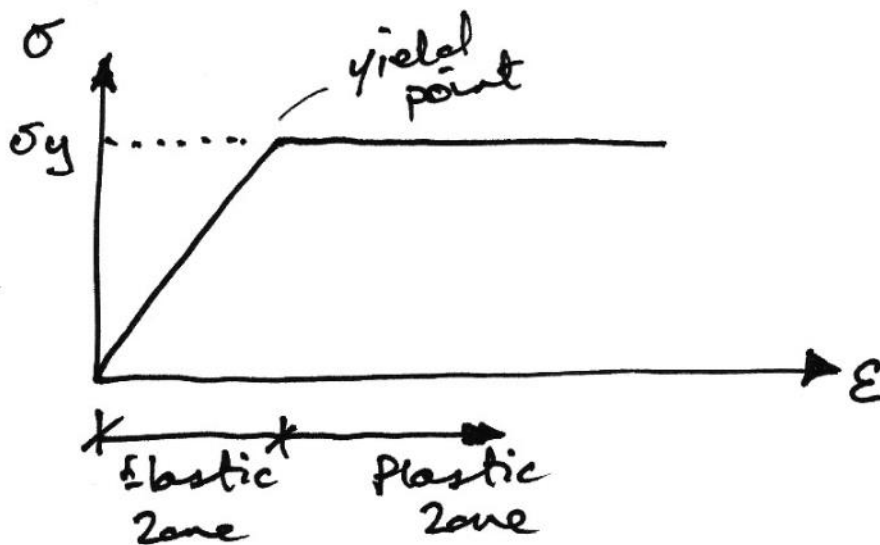
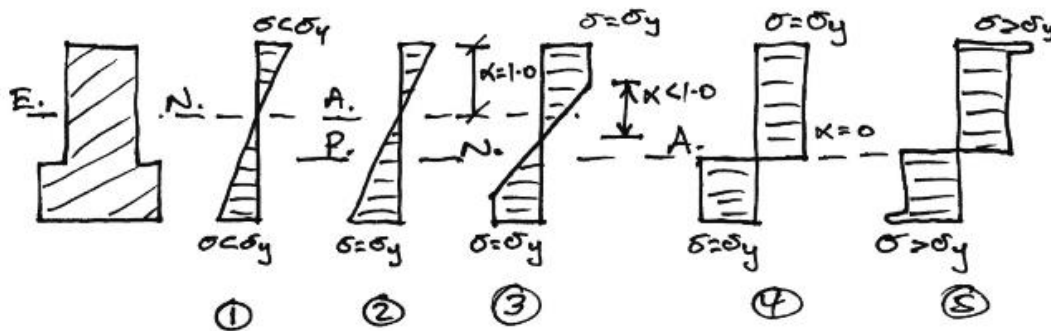


Figure 20- Idealized stress-strain diagram

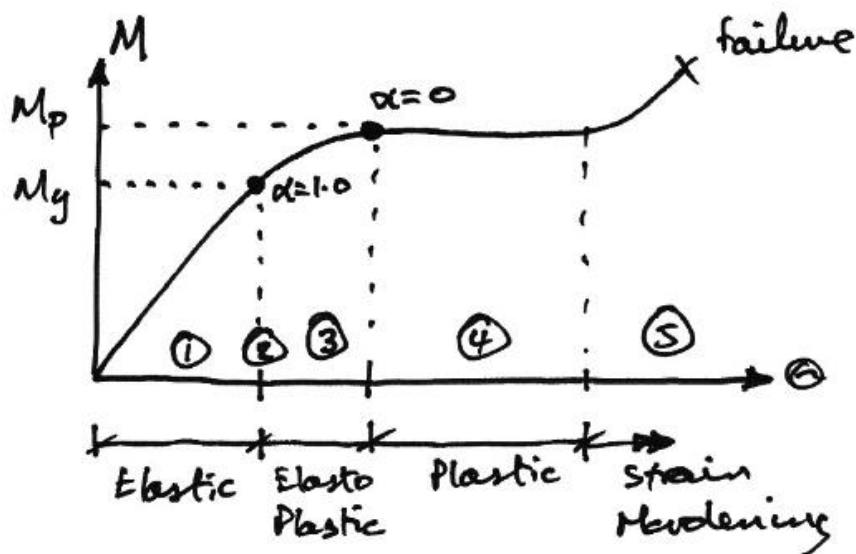
According to the idealized stress-strain diagram when the stress reaches the yield point the strain will continue till the infinity. Since so much post-yield strain is modeled, the actual material must also be able of allowing such strains. That is, it must be appropriately ductile for the idealized stress-strain curve to be useable. Next, we consider the behavior of a cross-section of an ideal elastic-plastic material expose to bending. In doing so, we require the relationship between applied moment and the rotation of a cross-section.

4-2-2 Cross Section Behavior

We consider an arbitrary cross-section with a vertical plane of symmetry, which is also the plane of loading. We consider the cross section subject to an increased bending moment and evaluate the stresses at each stage.



Cross-Section and Stresses



Moment-Rotation Curve

Figure 21- Cross section stresses and Moment-rotation curve

4-2-2-1 Stage 1 – Elastic Behavior

In the elastic behavior of the section the moment that applied to the section causes the stress that less than the yield stress of the materials.

4-2-2-2 Stage 2 – Yield Moment

At this this stage the moment that applied to the cross-section is enough to cause the yield stress in the outermost fiber of the cross-section. And other stresses of the cross-section are lower than the yield stress value. And in this stage the cross-section can be analysis and design elastically because all the fibers still in elastic limit. And the ratio of elastic to plastic is equal to one, $\alpha = 1.0$

4-2-2-3 Stage 3 – Elasto-Plastic Bending

At this stage the moment that applied to the cross section has been increased more than the yield moment. Since from the idealized stress-strain diagram the material cannot withstand a stress greater than yield stress, the fibers at the yield stress have moved inside towards the center of the beam. Therefore over the cross-section there is an elastic core and a plastic area. The ratio of the depth of the elastic core to the plastic region is $1 < \alpha < 0$ since more moment is being applied and no stress is bigger than the yield stress, extra rotation of the section occurs.

4-2-2-4 Stage 4 – Plastic Bending

The applied moment in this stage to the cross-section cause the yield stress in the all fiber of the materials. This defined as the Plastic Moment Capacity of the section and because of there are no fibers at an elastic stress, $\alpha = 0$. Also, note that the full plastic moment needs an infinite deformation at the neutral axis and so is physically impossible to achieve. However, it is closely approximated in practice. Any try at increasing the moment at this point simply results in more rotation, once the cross-section has sufficient ductility.

Therefore in steel members, the cross-section classification must be plastic and in concrete members, the section must be under-reinforced.

4-2-2-5 Stage 5 – Strain Hardening

Due to strain hardening of the material, a small amount of extra moment can be continued. The above moment-rotation curve represents the behavior of a cross-section of a regular elastic-plastic material. However, it is usually further simplified as follows:

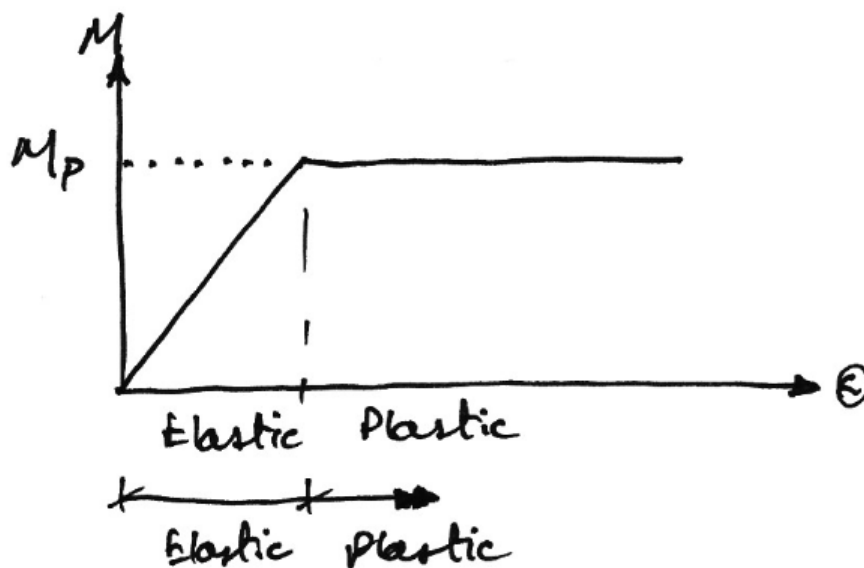


Figure 22-Idealized stress-strain diagram

With this idealized moment-rotation curve, the cross-section linearly stands moment up to the plastic moment capacity of the section and then yields in rotation an indeterminate amount. Again, to use this idealization, the actual section must be capable of sustaining large rotations – that is it must be ductile.

4-2-3 Plastic Hinge

Note that once the plastic moment capacity is reached, the section can rotate freely – that is, it behaves like a hinge, except with moment of M_p at the hinge. This is called a *plastic hinge* and is the basis for plastic analysis. At the point of plastic hinge, stresses stay constant, but strains and hence rotations can increase.

4-3 Analysis of Rectangular Cross Section

Since we know that a cross section can carry more load than just the yield moment, we are interested in how much this value. In other words, we want to find the elastic moment and plastic moment, and we do so for a rectangular section. Taking the stress diagrams from those of the moment-rotation curve examined previously, we have:

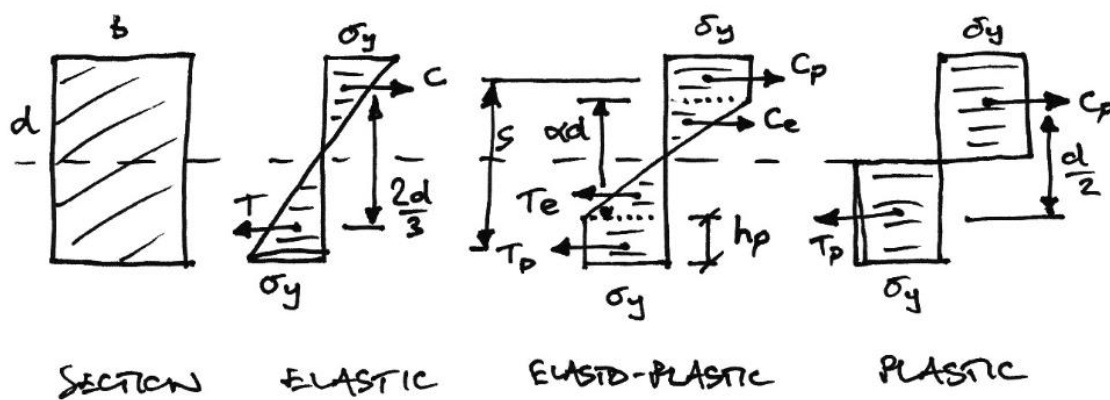


Figure 23-Stress distribution of a rectangular section in elastic, elasto-plastic, and plastic

4-3-1 Elastic Moment

From the diagram:

$$M_Y = C \cdot \frac{2}{3} d$$

But, the force (or the volume of the stress block) is:

$$C = T = \frac{1}{2} \sigma y \frac{d}{2} b$$

Hence

$$\begin{aligned} M_Y &= \left(\frac{1}{2} \sigma y \frac{d}{2} b \right) \left(\frac{2}{3} d \right) \\ &= \sigma y \frac{bd^2}{6} \\ &= \sigma y Z \end{aligned}$$

The term $bd^2/6$ is thus a property of cross section called the elastic section modulus and it is termed Z .

4-3-2 Elasto-Plastic Moment

The moment in the section is made up of plastic and elastic components:

$$M_{EP} = M'_E + M'_P$$

The elastic component is the same as previous, but for the reduced depth, $d\alpha$ instead of the overall depth, d :

$$\begin{aligned} M'_E &= \left(\frac{1}{2} \sigma_Y \frac{\alpha d}{2} \right) \left(\frac{2\alpha d}{3} \right) \\ &= \sigma_Y \cdot \alpha^2 \cdot \frac{bd^2}{6} \end{aligned}$$

The plastic component is:

$$M'_P = C_P \cdot s$$

The lever arm, s , is:

$$s = \alpha d + h_p$$

But

$$h_p = \frac{d - \alpha d}{2} = \frac{d}{2}(1 - \alpha)$$

Thus,

$$\begin{aligned} s &= \alpha d + \frac{d}{2} - \frac{\alpha d}{2} \\ &= \frac{d}{2}(1 + \alpha) \end{aligned}$$

The force is:

$$\begin{aligned} C_p &= \sigma_y h_p b \\ &= \sigma_y b \frac{d}{2}(1 - \alpha) \end{aligned}$$

Hence,

$$\begin{aligned} M_p' &= \left[\sigma_y b \frac{d}{2}(1 - \alpha) \right] \cdot \left[\frac{d}{2}(1 + \alpha) \right] \\ &= \sigma_y \frac{bd^2}{4}(1 - \alpha^2) \end{aligned}$$

And so the total elasto-plastic moment is:

$$\begin{aligned} M_{EP} &= \sigma_y \cdot \alpha^2 \cdot \frac{bd^2}{6} + \sigma_y \frac{bd^2}{4}(1 - \alpha^2) \\ &= \sigma_y \frac{bd^2}{6} \cdot \frac{(3 - \alpha^2)}{2} \end{aligned}$$

4-3-3 Plastic Moment

From the stress diagram:

$$M_p = C \times \frac{d}{2}$$

\And the force is:

$$C = T = \sigma_y \frac{d}{2} b$$

Hence:

$$\begin{aligned} M_p &= \left(\sigma_y \frac{bd}{2} \right) \left(\frac{d}{2} \right) \\ &= \sigma_y \cdot \frac{bd^2}{4} \\ &= \sigma_y \cdot S \end{aligned}$$

The term $bd^2/4$ is a property of cross section called the plastic section modulus and it is termed S.

Conclusions

- As a general most of materials shows the ductile and brittle behavior, this behavior is mainly referred to the composition of the materials, like steel, show brittle behavior when a high amount of carbon content and it is ductile when the Carbon content is reduced.
- Temperature has a significant effect on the ductile and brittle behavior of materials, which is at low-temperature materials become harder and more brittle, whereas at high temperature they become softer and more ductile.
- The structures that are designed with plastic behavior can carry more load an moment with large deformation, but the others that designed within elastic limit they carry less load and moment with the small amount of deformation, and when the load is removed it returns to it is original shape and size.
- The stress-strain diagram of specific materials is strongly dependent on the shape, size, of the specimen and the rate of loading during the test.
- The stress-strain diagram of specific materials is similar to each other but never will be same.
- Materials with ductile behavior can undergo a large deformation before fracture.
- The ultimate stress and fracture stress in the brittle materials is same, because brittle materials have no yield stress.

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