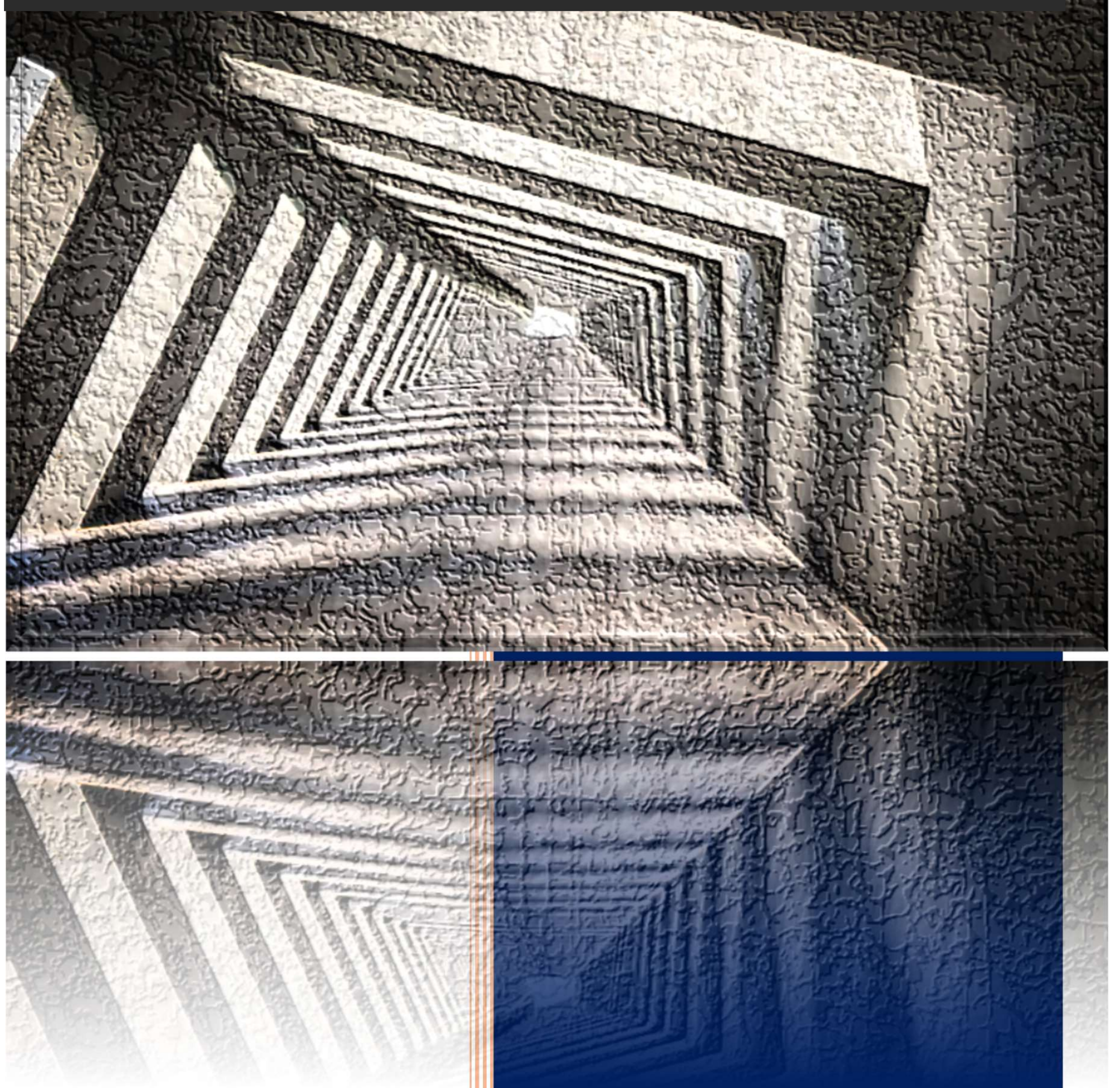
2022

BEAMS WITH OPENINGS DESIGN AND DETAILING



BY ENGINEER

Amanj Muhammad Faqy

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ABSTRACT

Ordinary beams with openings and deep beams with and without openings are considered disturbed regions where their strains within any section are significantly nonlinear. Therefore, it is not adequate to design those regions using either bending theory or conventional shear design equations. Hence, it is essential to rely on a rational method such as the strut-and-tie model.

The behavior of experimentally tested reinforced normal- and high-strength concrete simply supported shallow beams (with and without openings) and simple and continuous deep beams (with openings) was studied. In this study, the Strutand-Tie Models STM for all such selected beams are suggested based on the available experimental results of crack patterns, modes of failure, and internal stresses trajectors obtained from elastic finite element analysis. The obtained STM results are compared with test results.

To draw a complete picture of the response of the studied beams, a 3D nonlinear finite element analysis is conducted. From which, the output results of cracking patterns, deflections, failure mode and strain and stress distributions (that can not be obtained using the strut-and-tie model) are obtained

In addition, a full design procedure along with numerical examples, reinforcement detailing, and design recommendations for beams with openings only is presented.

INTRODUCTION

1.1 GENERAL

In practice, transverse openings in Reinforced Concrete, RC, beams are a facility, which allows the utility line to pass through the structure such as a network of pipes and ducts (which is necessary to accommodate essential services like water supply, sewage, air-conditioning, electricity, telephone, and computer network(, Fig. 1.1. Passing utility services through openings in the floor beam webs minimizes the required story height and encourages the designer to reduce the height of the structure, which leads to more economical design. Including transverse openings in the web of a reinforced concrete beam and therefore, the sudden changes in the dimensions of the cross section of the beam; the corners of the opening would be subjected to stress concentration and it is possible to induce transverse cracks in the beam. Also, it can reduce the stiffness, which lead to deformations and excessive deflections under service load and considerable distribution of forces and internal moments in a continuous beam. So, the effect of openings on the strength and behavior of reinforced concrete beams must be considered and the design of these beams needs special consideration. However, current codes of practice for design of RC structures do not provide provisions for design of RC beams with openings.

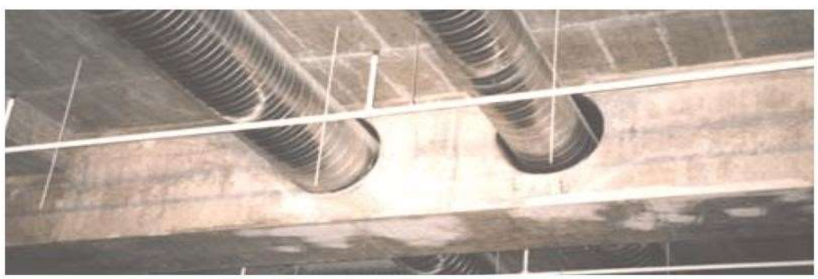


Figure 1.1 An ordinary beam with circular openings.

In this research, two types of reinforced concrete beams (ordinary and deep) with and without openings are studied. Reinforced concrete deep beams have useful applications in tall buildings, offshore structures, long-span structures (as transfer girders), foundations, and water tanks [Khalaf, (1986), Mahmoud, (1992)]. Since deep beams usually fail in shear at the ultimate limit state, their shear capacities have to be accurately understood. In continuous deep beams, the regions of high shear and high moment coincide and failure usually occurs in these regions. In simple deep beams, the region of high shear coincides with the region of low moment. Current codes, e.g. the ACI Code (2011) [3] and the Egyptian Code (2007) [10], define a beam to be deep when the span-to-overall member depth ratio (L/h) is less than or equal to 4, or the shear span-tooverall member depth ratio (a/h) is less than or equal to 2 and span-to-depth ratio (L/d) is less than or equal to 4, or the shear span-to-depth ratio (a/d) is less than or equal to 2, respectively. As a result of its proportions, the strength of a deep beam is usually controlled by shear, rather than by flexure, provided that normal amounts of longitudinal reinforcement are used. On the other hand, shear strength of deep beams is significantly greater than that predicted using expressions developed for shallow (ordinary) beams because deep beams have a more complex and different behavior in many features in comparison with ordinary beams:

- in deep beams, the hypothesis of Bernoulli is not valid; i.e., transverse sections which are plane before bending dose not remain plane after bending,
- the neutral axis does not usually lie at mid-depth and moves away from the loaded face of the member as the span-to-depth ratio decreases, and
- flexural stresses and strains are not linearly distributed across the beam depth [(Winter and Nilson (1978)].

Three design approaches are available for deep beams; namely, a semi-empirical design approach, a design approach based on stress analysis, and a strut-and-tie modeling. In design codes, the semi-empirical design approach is based on some empirical shear equations. In this approach, the concrete and steel reinforcement do not interact with each other, or in other words, this conventional approach does not give a physical representation of the interplay between concrete and steel contribution to shear strength. The second approach, based on stress analysis, involves the use and development of finite element models which consider the effect of cracking and transverse tensile strains on concrete behavior. Finally, a strut-and-tie model involves the development and design of an analogous truss. This contains concrete struts, tension ties, and nodal zones that realistically model the internal load path within the actual structure.

Openings in the web area of deep beams are frequently provided for essential services and accessibility, for example door openings, windows, ventilating ducts and heating pipes. Such openings may influence the beam behavior (either the ultimate capacity or the serviceability requirements) and stress distribution especially when openings are present in the critical shear zones and in the load path between the loading plate and the end support. The main factors affecting the behavior and ultimate capacity of deep beams with web openings are as follows:

- span-to-depth ratio (L/d),
- cross section properties,
- amount and location of main longitudinal reinforcement,
- amount, type, and location of web reinforcement,
- properties of concrete and reinforcements,
- shear-span-to-depth ratio (a/d),
- type and position of loading, and
- size, shape and location of web openings, etc…

Most current codes [3,10,12] give simplified design methods for deep beams without special consideration to the effect of web openings and no national codes even provide any guidance for the design of deep beams with openings. These design methods are based mainly on tests of deep beams constructed from Normal-Strength Concrete, NSC, with design compressive strength generally less than 50MPa. There have been extensive experimental and analytical investigations of simply supported deep beams with and without web openings. Very few tests of continuous deep beams constructed from NSC with and without web openings have been reported, while tests on reinforced

High Strength-Concrete HSC continuous deep beams with web openings have been rarely reported.

1.2 PROBLEM IDENTIFICATION

The use of reinforced concrete beams with openings is necessary to pass various services, and the analysis and design of such beams need special consideration. Conventional methods of analysis and design of solid beams cannot be used for beams with openings. As a result of the nonlinear character of the strains in openings' segments in shallow- and deep-beams as D-regions, a more suitable design method is needed. So, the strut-and-tie model has been used in order to obtain the optimal model of the load path. This can be achieved with the aid of linear elastic finite element analysis to obtain the stress trajectories.

The strut-and-tie model has been proved to be a useful and consistent method for the analysis and design of structural concrete including, of course, D-regions. This model is an extension of the so-called truss analogy, and gives a physical representation of the actual stress fields resulting from applied loads and support conditions. In this method the flow of forces in a structural member is approximated by the use of struts to represent the flow of compressive stresses and ties to represent the flow of tensile stresses.

1.3 RESEARCH SIGNIFICANCE

This thesis aims not to present but to revise the design and detailing of reinforced concrete (shallow and deep) beams with and without openings, utilizing both of the strut-and-tie model and a 3-D nonlinear finite element analysis. The concept of strut-and-tie model is introduced and extended to include several types of reinforced concrete simple and continuous shallow and deep beams that are subject to top point loads. The effect of the shear span-to-depth ratio, the concrete strength, opening size and shape, the variation of mechanisms between shallow and deep beams, and simple versus continuous beams, are considered in design.

In addition, the finite element package (ANSYS-12) [5] is used to carry-out the 3-D nonlinear finite element analysis. This finite element analysis, on one hand, is used to check the output results that are obtained using the strut-and-tie model and completes, on the other hand, the understanding of the behavior of the considered reinforced concrete beams (shallow and deep). The verification process of both the strut-and-tie results and the finite element model is also achieved using the experimental data available in literature. The key features (the shear span-to-depth ratio, span-to-depth ratio, concrete strength, opening size and shape, load type, and web reinforcement) that affect the design and detailing of normal- and high-strength reinforced concrete beams utilizing the 3-D nonlinear finite element analysis are presented. Proposed design procedure for beams with openings and design recommendations are introduced.

1. 4 OBJECTIVES AND SCOPE

The purpose of this study is to investigate the effect of web openings on the behavior of reinforced concrete shallow and deep beams, simple and continuous considering the effects of different parameters such as size, shape and location of openings, web reinforcement, shear spanto-depth ratio and concrete strength. This study focuses on the design and detailing of Reinforced Concrete, RC, shallow and deep beams with and without openings utilizing both:

- the Strut-and-Tie Models STM and
- a 3-D nonlinear finite element analysis using ANSYS-package [5].

The verification process of both the strut-and-tie results and the finite element model is also achieved using the experimental data available in literature.

DESIGN PROCEDURE , DETAILING , AND DESIGN RECOMMENDATIONS FOR BEAMS WITH OPENINS

2.1 INTRODUCTION

The aim of this chapter is to introduce the design steps through which the amount and details of reinforcement around openings in reinforced concrete (shallow and deep) beams can be determined.

2.2 SHALLOW (ORDINARY) BEAMS

In this chapter, two types of reinforced concrete (shallow and deep) beams with openings are studied. Some of the existing design codes; e.g., the ACI 318M-11 Code [3] and the Egyptian Code (2007) [10], define a beam to be **shallow** when:

1. ACI 318M-11 Code [3]:
   - ℓ_n /h > 4 or
   - a/h > 2.
2. Egyptian Code (2007) [10]:
   - L/d > 4 or
   - a/d > 2.

where L and ℓ_n is the effective span and clear span, respectively.

2.2.1 General Guidelines

A review to the literature on the behavior and strength of beams with web openings and refereeing to Fig. 5.1 the following guidelines can be used to facilitate the selection of the size and location of web openings:

- For Tee beams, openings should preferably be positioned flushed with the flange for ease in construction. In the case of rectangular beams, openings are commonly placed at mid-depth of the section, but they may be placed eccentrically with respect to the depth if situation dictates. Care must be exercised to provide sufficient concrete cover to the reinforcement for the chord member above and below the opening. The compression chord should also have sufficient concrete area to develop the ultimate compression block in flexure and have sufficient depth to provide effective shear reinforcement.
- Openings should not be located closer than half the beam depth, 0.5h, to the supports to avoid the critical region for shear failure and reinforcement congestion. Similarly, positioning of an opening closer than 0.5h to any concentrated load should be avoided.
- The depth of openings should be limited to 50% of the overall beam depth.
- The factors that limit the length of an opening are the stability of the chord member, (in particular the compression chord), and the serviceability requirement of deflection. It is preferable to use multiple openings providing the same passageway instead of using a single long opening.
- When multiple openings are used, the width of post separating two adjacent openings should not be less than 0.5h or 100mm, whichever is larger, to ensure that each opening behaves independently.

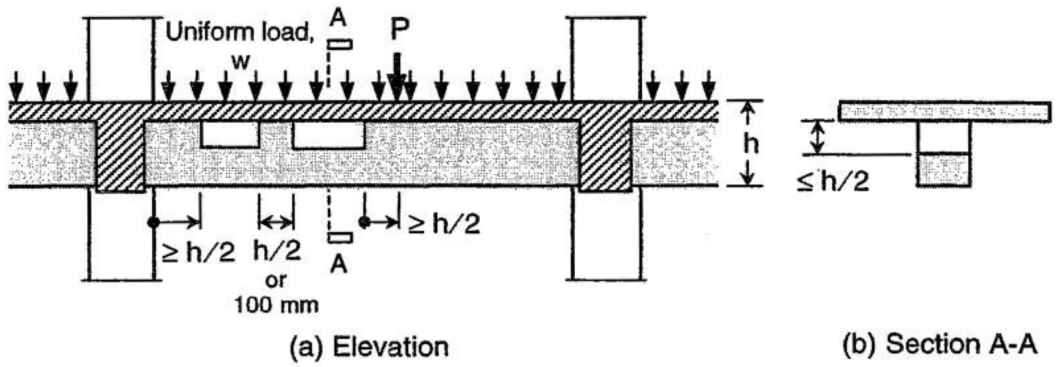


Figure 5.1 Guidelines for the location of web openings (Tan et al., 1996).

2.2.2 Design of Reinforced Concrete Beams with Small Openings Using the Traditional Approach

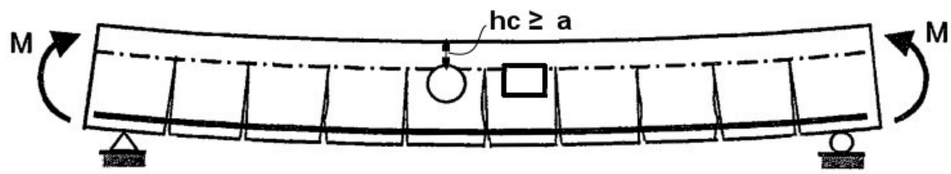
Openings that are circular, square, or nearly square in shape may be considered as small openings provided that the depth (or diameter) of the opening is in a realistic proportion to the beam size, say, about less than or equal to 0.40 times the overall beam depth. In such a case, the beam action may be assumed to prevail. Therefore, analysis and design of a beam with small openings may follow the similar course of action as that of a solid beam. The provision of openings, however, produces discontinuities or disturbances in the normal flow of stresses, thus leading to stress concentration and early cracking around the opening region. Similar to any discontinuity, special reinforcement, enclosing the opening close to its periphery, should therefore be provided in sufficient quantity to control crack widths and prevent possible premature failure of the beam.

2.2.2.1 Pure Bending

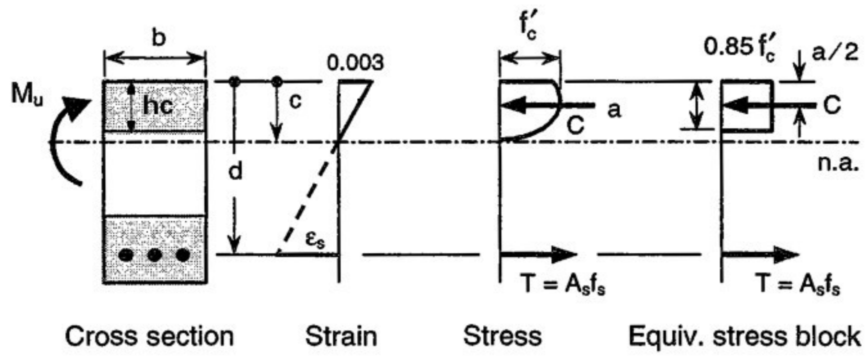
In the case of pure bending, placement of an opening completely within the tension zone does not change the load-carrying mechanism of the beam because concrete there would have cracked anyway in flexure at ultimate, see Fig. 5.2. Mansur and Tan [26] have illustrated this through worked out examples, supported by test evidence. Thus, the ultimate moment capacity of a beam is not affected by the presence of an opening as long as the minimum depth of the compression chord, h_c is greater than or equal to the depth of the ultimate compressive stress block, a, that is, when

$$h_c \geq a = \frac{A_s f_y}{0.85 f_c' b} \tag{5.1}$$

in which A_s is the area of tensile reinforcement, f_y is the yield strength of tensile reinforcement, f_c' is the cylinder compressive strength of concrete, and b is the width of the compression zone. However, due to reduced moment of inertia at a section through the opening, cracks will initiate at an earlier stage of loading. In spite of this, the effects on maximum crack widths and deflection under service load have been found to be only marginal, and may safely be disregarded in design.



(a) Beam with small openings.

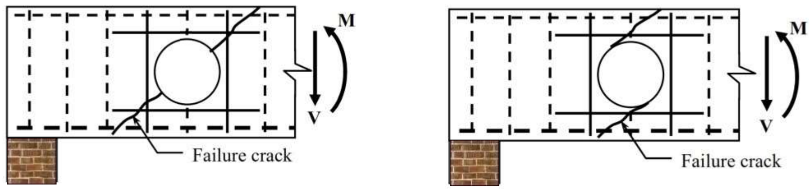


(b) Condition through the opening at collapse.

Figure 5.2 Beam with opening under pure bending.

2.2.2.2 Combined Bending and Shear

According to the traditional design philosophy, bending moment and shear force are treated separately. A section subject to combined bending and shear, therefore, is designed first for bending, and second for shear. In a beam, shear is always associated with bending moment, except for the section at inflection point. When a small opening is introduced in a region subjected to predominant shear and the opening is enclosed by reinforcement, as shown by solid lines in Fig. 5.3, test data reported by Hanson (1969), Somes and Corley (1974), Salam (1977), and Weng (1998) indicate that the beam may fail in two distinctly different modes. The first type is typical of the failure commonly observed in solid beams except that the failure plane passes through the center of the opening (Fig. 5.3a). In the second type, formation of two independent diagonal cracks, one in each member bridging the two solid beam segments, leads to the failure (Fig. 5.3b). Labeled respectively as beam-type failure and frame-type failure (Mansur, 1998), these modes of failure require separate treatment.



(a) Beam-type failure (b) Frame-type failure

Figure 5.3 The two modes of shear failure around small openings.

Similar to the traditional shear design approach, it may be assumed in both of the two cases that the nominal shear resistance, V_n, is provided partly by the concrete, V_c, and partly by the shear reinforcement crossing the failure plane, V_s. That is,

$$V_n = V_c + V_s \tag{5.2}$$

Design for Bending

It is preferred to put the opening close to the support where the bending moment is small designing for bending may be carried out independently, in the usual manner, and they combined the results to the shear design solutions. Design for bending will be for two sections; the first is at maximum bending at mid-span of beam using an effective depth d, and the second section is through the center of opening or the flexural design of the section at the center of opening always requires a similar amount of the tension reinforcement in the mid-span of the beam.

Shear Design for Beam-type Failure

In designing for beam-type failure, a 45^oinclined failure plane, similar to a solid beam may be assumed, the plane being traversed through the center of the opening as shown in Fig. 5.4. Following the simplified approach of the ACI Code [3], the shear resistance V_c by the concrete for solid beams follows the following simple equation:

$$V_c = \frac{1}{6}\sqrt{f_c'}b_w d \tag{5.3}$$

When the beam contains a small opening, Mansur (1998) proposed that the term d in Eq. (5.3) be replaced by the net depth, $(d - d_o)$, so the shear resistance V_c is given by:

$$V_c = \frac{1}{6}\sqrt{f_c'}b_w\,(d - d_o) \tag{5.4}$$

in which f_c' is the concrete cylinder compressive strength in MPa, b_w is the width of the web in mm, d is the effective depth in mm, and d_o is the diameter (or depth) of opening in mm. Equation (5.4) is applicable for a beam made up of normal-weight concrete. For light-weight concrete beams, an average reduction factor of 0.8 may be assumed, as suggested by the ACI Code [3].

For the contribution of the shear reinforcement, V_s, reference may be made to Fig. 5.4. It may be seen that the stirrups available to resist shear across the failure plane are those by the sides of the opening within a distance $(d_v - d_o)$, where d_v is the distance between the top and bottom longitudinal rebars, and d_o is the diameter (or depth) of opening, as shown in the figure. The contribution of diagonal reinforcement, if any, intercepted by the failure plane may also be taken into account in the calculation of shear resistance. Thus,

$$V_s = V_{sv} + V_{sd} = \frac{A_v f_{yv}}{S}(d_v - d_o) + A_d f_{yd} \sin\alpha \tag{5.5}$$

in which V_{sv} and V_{sd} are the contribution of vertical and diagonal reinforcement, respectively; A_v is the area of vertical legs of stirrups per spacing S across the failure plane; A_d is the total area of diagonal reinforcement through the failure surface; α is the inclination of diagonal reinforcement; f_{yv} is the yield strength of stirrups; and f_{yd} is the yield strength of diagonal reinforcement.

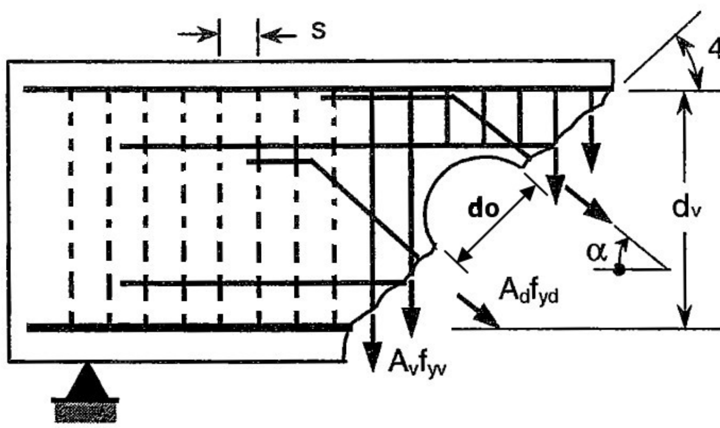


Figure 5.4 Shear resistance, V_s, provided by shear reinforcement at an opening.

Knowing the values of V_c and V_s, the required amount of web reinforcement to carry the factored shear through the center of the opening may be calculated in the usual way. This amount should be contained within a distance $(d_v - d_o)/2$, or preferably be lumped together on either side of the opening. Other restrictions applicable to the usual shear design procedure of solid beams must also be strictly adhered to.

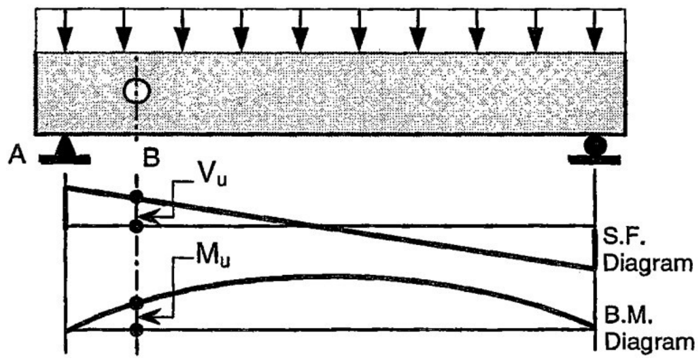
Shear Design for Frame-type Failure

Frame-type failure occurs due to the formation of two independent diagonal cracks, one on each of the chord members above and below the opening, as shown in Fig. 5.3b. It appears that each member behaves independently similar to the members in a framed structure. Therefore, each chord member requires independent treatment, as suggested by Mansur (1998).

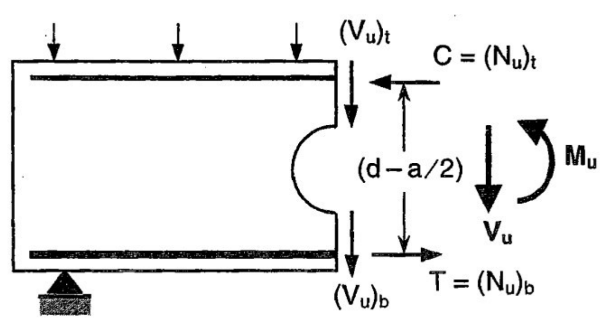
In order to design reinforcement for this mode of failure, let us consider the free-body diagram at beam opening, Fig. 5.5. Clearly, the applied factored moment, M_u, at the center of the opening from the global action is resisted by the usual bending mechanism, that is, by the couple formed by the compressive and tensile stress resultants, N_u, in the members above and below the opening. These stress resultants may be obtained from:

$$(N_u)_t = \frac{M_u}{\left[d - \frac{a}{2}\right]} = -(N_u)_b \tag{5.6}$$

subject to the restrictions imposed by Eq. (5.1). In this equation, *d* is the effective depth of the beam, *a* is the depth of equivalent rectangular stress block, and the subscripts *t* and *b* denote the top and bottom cross members of the opening, respectively.



(a) Beam and loading.



(b) Beam segment A-B.

Figure 5.5 Free-body diagram at beam opening.

The applied shear, V_u, may be distributed between the two members in proportion to their crosssectional areas according to Nasser et al., (1967). Thus,

$$(V_u)_t = V_u \left[\frac{A_t}{A_t + A_b}\right] \tag{5.7a}$$

and

$$(V_u)_b = V_u - (V_u)_t \tag{5.7b}$$

Knowing the factored shear and axial forces, each member can be independently designed for shear by following the same procedure as for conventional solid beams with axial compression for the top chord and axial tension for the bottom chord.

2.2.2.3 Reinforcement Detailing

The reinforcement details for the solid segments of the beam should follow the normal detailing procedure for simple and continuous beams. The opening segment requires additional reinforcement and it should be detailed carefully keeping in mind the strength and crack control requirements.

Consideration of beam-type failure will require long stirrups to be placed on either side of the opening, while that of the frame-type failure will need short stirrups above and below the opening. For anchorage of short stirrups, nominal bars must be placed at each corner, if none is available from the design of solid segments. This will ensure adequate strength. For effective crack control, nominal bars should also be placed diagonally on either side. The resulting arrangement of reinforcement around the opening is shown in Fig. 5.6. Under usual circumstances, introduction of a small opening with proper detailing of reinforcement does not seriously affect the service load deflection. However, in case of any doubt one can follow the procedure described for beams with large openings to calculate the service load deflections and check them against the permissible values.

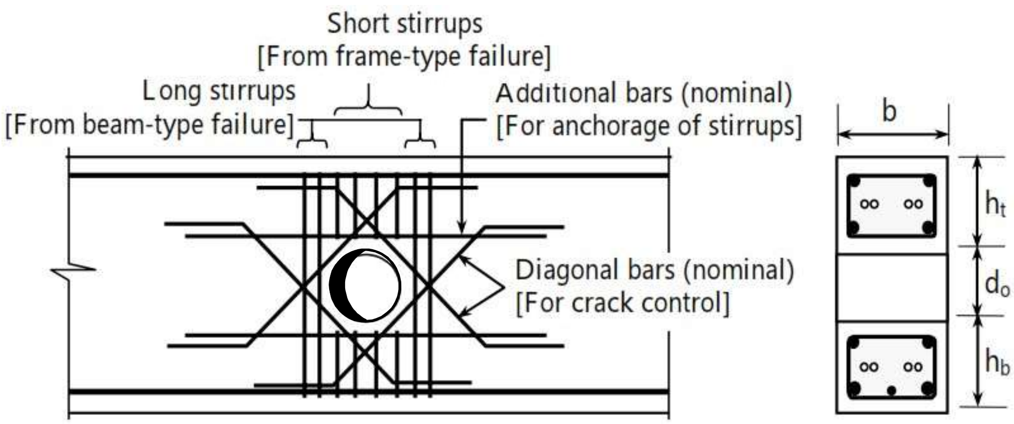


Figure 5.6 Reinforcement details around a small opening.

2.2.2.4 Numerical Example for Beam ND80X350 (Case 2) with Small Openings

A simply supported rectangular beam 125mm wide and 250mm height carries two concentrated and symmetrical factored loads 50kN on a span of 1.60m, Fig. 5.7. The beam contains a 80mm diameter circular opening located at mid-depth of the beam and at a distance of 350mm from the left support. It is required to design the shear reinforcement for the opening segment of the beam.

Given $f_c' =$, $f_y =$, and $f_{yv} =$

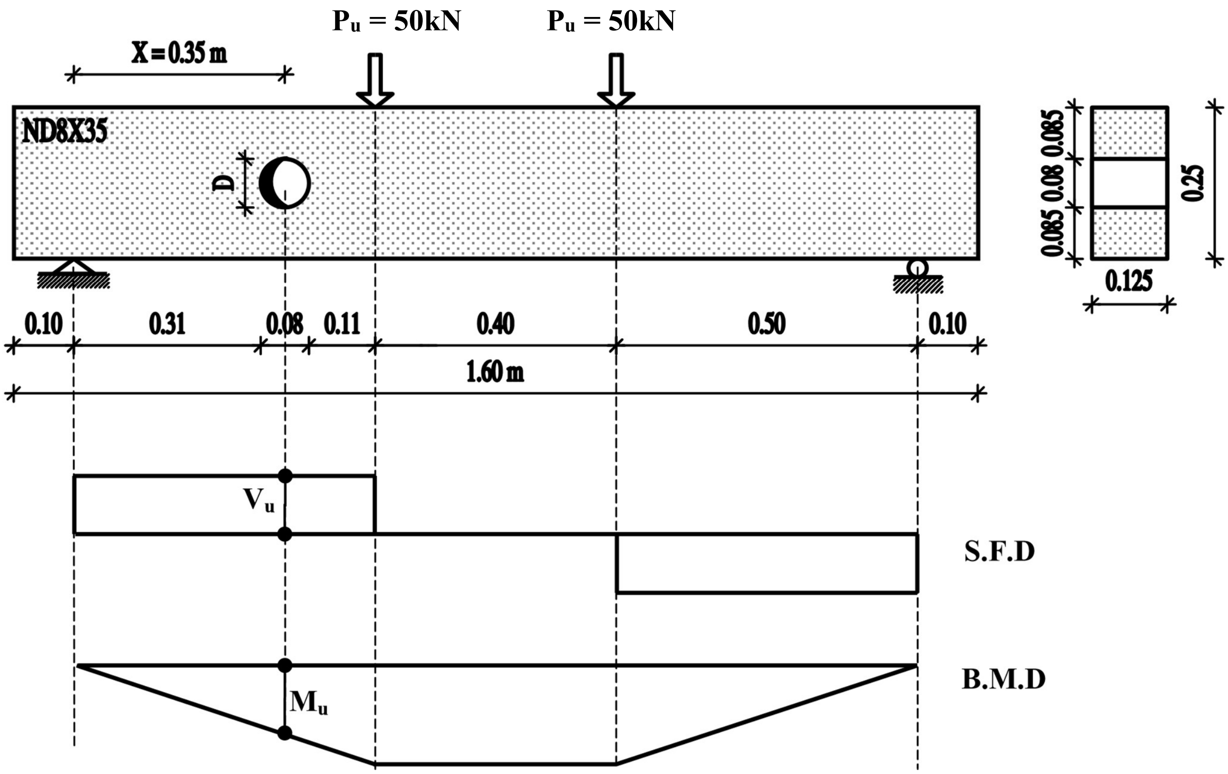


Figure 5.7 Beam ND80X350 and loading.

Solution:

(a) Analysis

At the center of opening, we get, Vu = 50kN and Mu = 17.50kN-m

Since the beam is subjected to combined bending and shear, Eqs. (5.7a) and (5.7b) will be used and give:

$$(V_u)_t = (V_u)_b = V_u \left[\frac{A_t}{A_t + A_b}\right] = 50 \left[\frac{85 \times 125}{85 \times 125 + 85 \times 125}\right] = 25kN$$

(b) Design for flexure

Flexural design of the section at mid-span:

$$M_u = 0.85 f_c' ba \times \left[d - \frac{a}{2}\right] \quad (5.8)$$

$$25 \times 10^6 = 0.85 \times 28.93 \times 125 \times a \times \left[217 - \frac{a}{2}\right]$$

We get $a = 41.44\text{mm}$

$$T = C$$

$$A_s f_y = 0.85 f_c' ba \quad (5.9)$$

$$A_s \times 450 = 0.85 \times 28.93 \times 125 \times 41.44$$

$$A_s = 283.10\text{mm}^2 \text{ (use } 2\Phi 14)$$

The flexural design of the section at the center of opening always requires a similar amount of the tension reinforcement in the mid-span of the beam, 2Φ14. Provide 2ϕ6 bars on the compression side of the beam for anchorage of stirrups.

(c) Shear design for beam-type failure

Assuming 20mm clear concrete cover andΦ6mm bars for stirrups,

$$d = t - Cover = 250 - (20 + 6 + 14/2) 217 mm$$

$$d_v = 250 - (2 \times 20 + 2 \times 6 + 6/2 + 14/2) = 188\ mm$$

Check the section adequacy:

Maximum shear. In order to ensure yielding of steel reinforcement when the failure strength in shear is reached and, hence, avoid web crushing failure, the upper limit of the factored shear is given by:

$$[V_u]_{max} = 5\phi V_c = 5\phi \times \tfrac{1}{6}\sqrt{f_c'}\, b_w\, (d - d_o) \quad (5.10)$$

In the ACI [3] Strength Design Method for shear, it is required that

$$V_u \leq [V_u]_{max} \quad (5.11)$$

where V_u is the factored shear force and is the strength reduction factor (0.85 for shear). From Eq. (5.10) we get,

$$[V_u]_{max} = 5 \times 0.85 \times \frac{1}{6}\sqrt{28.93} \times 125\ (217 - 80) \times 10^{-3} = 65.24 kN$$

which is greater than $V_u = 50\text{kN}$, therefore, the section is adequate.

Design of full-depth stirrups:

Maximum spacing of stirrups. For shear reinforcement to function effectively without any localized failure before the load-carrying mechanism is fully established, some limitations on maximum spacing of stirrups must be followed. Assuming that V_c is given by Eq. (5.4), ACI Code [3] limits the stirrup spacing to:

$$S_{max} = \frac{d}{4} \leq 600\text{mm} \qquad (0.5\phi V_c < V_u \leq 3\phi V_c) \quad (5.12)$$

$$S_{max} = \frac{d}{4} \leq 300\text{mm} \qquad (3\phi V_c < V_u \leq 5\phi V_c) \quad (5.13)$$

$V_u = 50\text{kN} > 3\,V_c = 39.15\text{kN}$ and less than 5 V_c=65.24kN, hence shear reinforcement is required, and $S_{max} = \frac{d}{4} = \frac{217}{4} = 54.25\text{mm} < 300\text{mm}$. Since $\phi V_s = V_u - \phi V_c = 50 - 0.85 \times 15.35 = 36.95\text{kN}$,

$$V_s = \frac{36.95}{0.85} = 43.47\text{kN}$$

Assuming that the shear resistance of the steel is provided by vertical stirrups only and that twolegged ϕ6mm stirrups (2-legs) are used, the required number of stirrups, *n*, is given by:

$$n = \frac{V_s}{A_v f_{yv}} = \left[\frac{43.47}{\left(2 \times \frac{\pi 6^2}{4}\right) \times 250}\right] 10^3 = 3.07 \cong 4$$

using of two full-depth stirrups on either side of the opening at a spacing of 50mm and positioning small stirrups above and below the opening, ,$(d_v - d_o)/2$, would satisfy the requirements of maximum spacing. In addition, nominal diagonal bars for crack control to be provided.

(d) Shear design for fram-type failure

Member below the opening (tension member):

For this section, $d = 85 - (20 + 14/2) = 58\text{mm}$; therefore,

$$[V_u]_{max} = 5 \times 0.85 \times \frac{1}{6}\sqrt{28.93} \times 125\ (58) \times 10^{-3} = 27.62kN$$

Which is greater than $(V_u)_b = 25kN$ and, thus, the section is adequate to avoid diagonal compression failure.

Neglecting the contribution of concrete and using two-legged stirrups of ϕ6mm bars will yield

$$V_u = \phi V_s = \phi\left(\frac{A_v f_{yv} d}{S}\right) \qquad (5.14)$$

$$S = \frac{\phi A_v f_{yv} d}{V_u} = \frac{0.85 \times 56.55 \times 250 \times 58}{25 \times 10^3} = 28\text{mm}$$

As V_u is less than $3\,V_c = 39.15\text{kN}$, the maximum S is $d/4 = 14.5\text{mm}$ which is quite small. Considering the difficulty in achieving proper compaction of concrete and keeping in mind that diagonal bars for crack control would resist part of the applied shear, it is decided to use 5 short stirrups below the opening in between the full-depth stirrups, which gives a spacing of about 15mm. Provide two nominal ϕ6mm longitudinal bars just below the opening for anchorage of stirrups.

Member above the opening (compression member):

Since the section for this member having identical dimension to that of the section below, and it is subjected to axial compression, the same spacing of stirrups can used and it would provide a conservative design and avoid any confusion during construction.

(e) Design for crack control

The reinforcement designed as above would ensure adequate strength. However, due to sudden reduction in beam cross section, stress concentration occurs at the edge of the opening. Adequate reinforcement with proper detailing should therefore be provided to prevent wide cracking under service load conditions.

In the case of small openings, the reinforcement requirements for crack control are quite less. Since full-depth stirrups are already provided by the sides of the opening to ensure adequate strength, provision of diagonal reinforcement may be considered to restrict the growth of cracks along the failure plan. An amount of diagonal reinforcement that is sufficient to carry the total shear along the 45^0failure plane (beam-type failure) has recently been recommended by Mansur (1998). Thus, the total area of diagonal reinforcement, $A_{d,}$ through the failure surface (Fig. 5.4) is

$$A_d = \frac{V_u}{\phi f_{yv} \sin \alpha} \tag{5.15}$$

in whichα is the inclination of diagonal reinforcement and f_{yv} is the yield strength of diagonal reinforcement. This amount should be distributed equally on either side of the opening and be placed perpendicular to this reinforcement to avoid confusion during construction and to take care of any possible load reversal.

For this example, use diagonal reinforcement to avoid crack control only under service load condition. Using Eq. (5.15), and assuming $f_{yd} = 450\text{MPa}$, the required area of diagonal reinforcement is

$$A_d = \frac{V_u}{\phi f_{yv} \sin \alpha} = \frac{50 \times 10^3}{(0.85 \times 450 \times \sin 45^0)} = 184.86\text{mm}^2$$

Use 4Φ8 diagonal bars in each direction.

(f) Reinforcement details

The final arrangement of reinforcement in the opening region of the beam is shown in Fig.5.8.

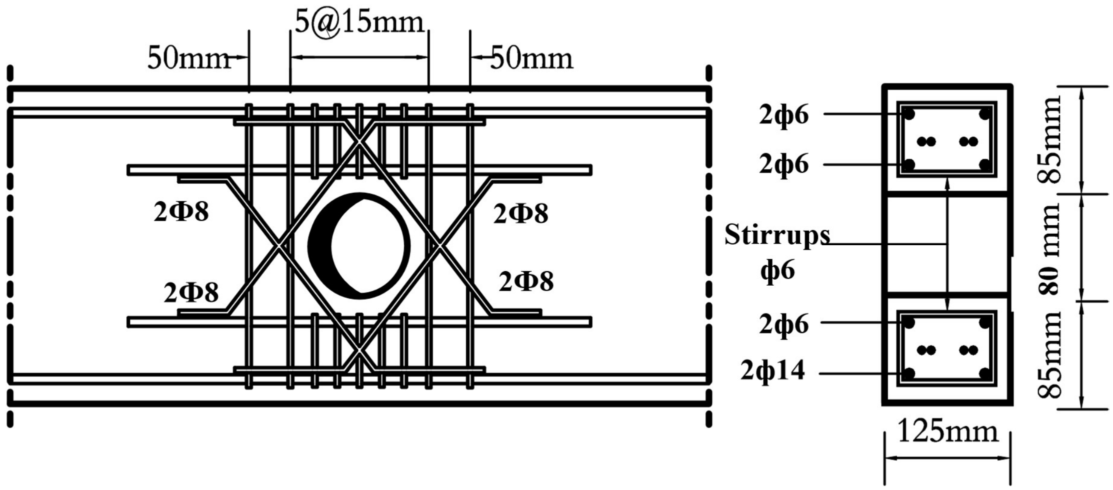


Figure 5.8 Reinforcement details of the Beam ND80X350.

2.2.3 Redesign of Reinforced Concrete Beam ND80X350 using Strut-and-Tie Method

The first step in this method is to visualize the flow of forces from the applied loads to the supports. This is accomplished from the obtained elastic principal stress trajectories from finite element analysis. The finite elements with compression stress trajectories will be replaced by compression elements (Struts) and the finite elements with tension stress trajectories will be replaced by tension elements (Ties). Once the model is obtained, the forces in the struts and ties can be calculated from statics. The required area of tension tie reinforcement is then chosen.

(a) Develop the strut-and-tie model

A Strut-and-Tie Model for the Beam ND80X350 in this example is shown in Fig. 5.9. Here,

the compressive struts are shown in dotted lines while tension ties are shown in solid lines.

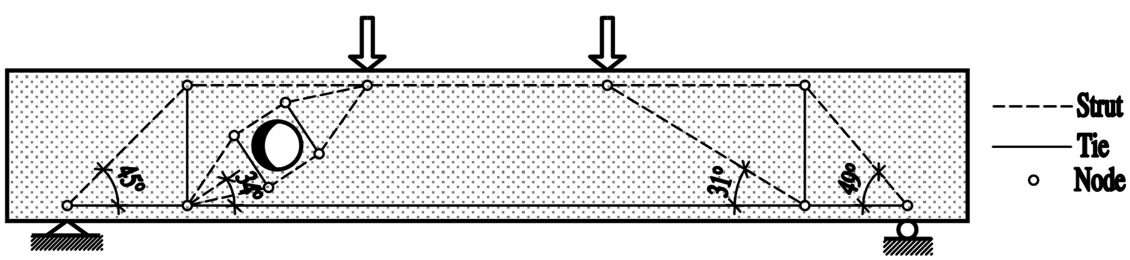


Figure 5.9 Proposed strut-and-tie model for Beam ND80X350.

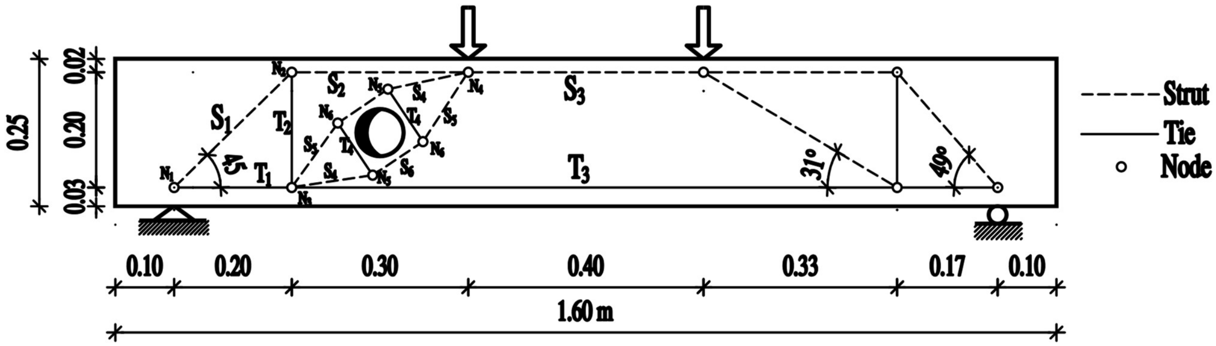


Figure 5.10 Details of the strut-and-tie model for Beam ND80X350.

(b) STM forces

The forces in all members are determined from statics and their magnitudes in kN are as indicated in Table 5.1. The struts, ties, and nodes are labeled as in Fig. 5.10. From equilibrium,

$$M_u = S_3 \times y_{ct} = 0.85 f_c' ba \times \left[d - \frac{a}{2}\right]$$

$$25 = 0.85 \times 28.93 \times 125 \times \mathrm{a} \times \left[217 - \frac{a}{2}\right]$$

$$\text{We get a} = 41.44\text{mm}$$

$$S_3 = T_3 = \frac{M_u}{y_{ct}} = \frac{25 \times 10^3}{\left[217 - \frac{41.44}{2}\right]} = 127.40kN$$

<u>Node 1:</u>

$S_1 = \frac{P_u}{sin\,\alpha_1} = \frac{50}{sin\,45^0} = 70.71$ kN

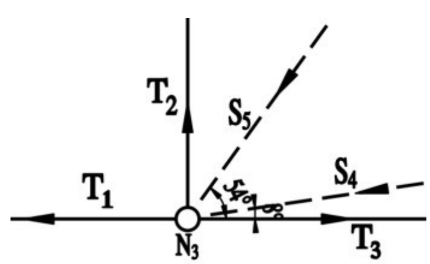
$T_1 = S_1\, cos\,\alpha_1 = 70.71 \times cos\,45^o = 50$ kN

<u>Node 2:</u>

$S_2 = S_1\, cos\,\alpha_1 = 70.71 \times cos\,45^o = 50$ kN

$T_2 = S_1\, sin\,\alpha_1 = 70.71 \times sin\,45^o = 50$ kN

<u>Node 3:</u>



$T_3 = T_1 + S_4\, cos\,\alpha_4 + S_5\, cos\,\alpha_5$

$127.40 = 50 + S_4\, cos\,8^o + S_5\, cos\;54^o$

$77.40 = S_4\, cos\,8^o + S_5\, cos\;54^o$ ……… (1)

$T_2 = S_4 \, sin \, \alpha_4 + S_5 \, sin \, \alpha_5$

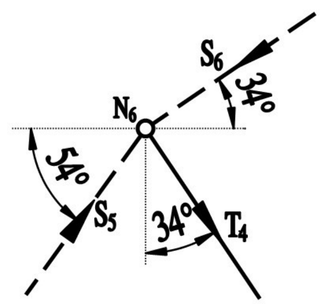
$50 = S_4 \, sin \ 8^o + S_5 \, sin \ 54^o$……… ( )

Solving Eqs. 1 and 2 yields:
$S_4 = 46.20\text{kN} \; and \; S_5 = 53.86\text{kN}$

Node 6:

$S_6 \, cos \, \alpha_6 - T_4 \, sin \, \alpha_4 = S_5 \, cos \, \alpha_5$
$S_6 \, cos \, 34 - T_4 \, sin \, 34 = 53.86 \, cos \, 54 \ldots\ldots\ldots. (1)$

$S_6 \, sin \, \alpha_6 + T_4 \, cos \, \alpha_4 = S_5 \, sin \, \alpha_5$
$S_6 \, sin \, 34 + T_4 \, cos \, 34 = 53.86 \, sin 54 \ldots\ldots\ldots (2)$



Solving Eqs. 1 and 2 gives:
$S_6 = 50.64\text{kN} \; and \; T_4 = 18.40\text{kN}$

Table 5.1 STM forces.

Member	**Force, kN**
S_1	70.71
S_2	50.00
S_3	127.40
S_4	46.20
S_5	53.86
S_6	50.64
T_1	50.00
T_2	50.00
T_3	127.40
T_4	18.40

(c) Reinforcement sizes: a-

Longitudinal reinforcement

<u>Bottom chord member</u>

For the bottom reinforcement

$$T_3 = S_3$$
$$As_3 f_y = S_3$$
$$A_s \times 450 = 127.40$$

$$A_s = \frac{127.40 \times 10^3}{450} = 283mm^2 \rightarrow \text{Use } 2\Phi14$$

Since curtailment of reinforcement is not possible before reaching the throat section, the same reinforcement is continued throughout the length of the bottom chord of opening segment.

<u>Top chord member</u>

Assuming that concrete will carry all of the compressive force, use compression reinforcement $2\phi6$ only to anchor vertical stirrups.

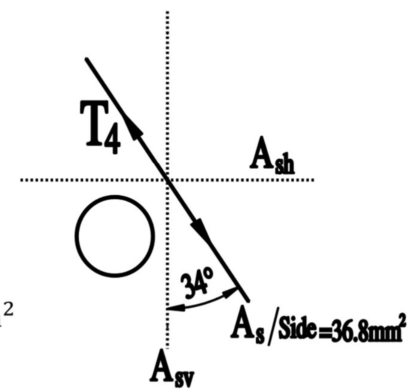
b- Transverse reinforcement

$$A_s f_y = T_4$$

$$\mathrm{A_s} \times 250 = 18.40 \times 1000$$

$$\mathrm{A_s} = \frac{18.40 \times 10^3}{250} = 73.60\mathrm{mm}^2$$

$$\mathrm{A_s/side} = 73.60/2 = 36.80\mathrm{mm}^2$$



$$\mathrm{A_{sv}} = \mathrm{A_s/side} \times \cos\alpha_4 = 36.80 \times \cos 34 = 30.51\mathrm{mm}^2$$

Use vertical stirrups $2\phi6$ at each side of opening.

$$\mathrm{A_{sh}} = \mathrm{A_s/side} \times \sin\alpha_4 = 36.80 \times \sin 34 = 20.58\mathrm{mm}^2$$

Use horizontal reinforcement $1\phi6$ at each side of opening.

Design for crack control

In this example, add diagonal reinforcement for crack control only under service load condition. Using Eq. (5.15) and assuming that $f_{yd} = 450\mathrm{MPa}$, the required area of diagonal reinforcement is

$$A_d = \frac{V_u}{\phi f_{yv} \sin \alpha} = \frac{50 \times 10^3}{(0.85 \times 450 \times \sin 45^0)} = 184.86\text{mm}^2$$

ΦUse 4 8 diagonal bars in each direction.

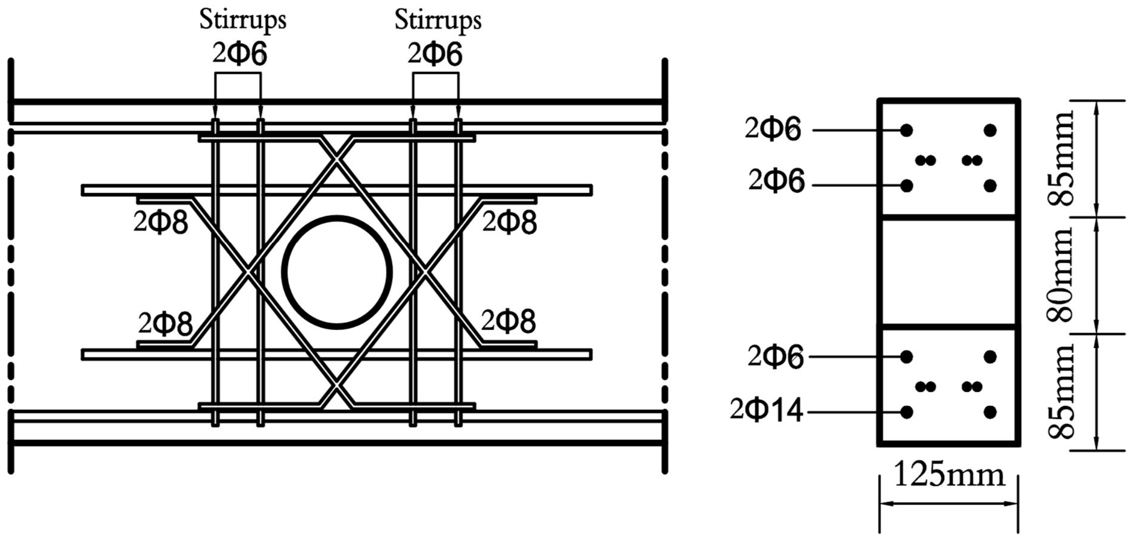


Figure 5.11 Reinforcement details of the Beam ND80X350 using strut-and-tie method.

5.2.4 Design of Reinforced Concrete Beams with Large Openings Using the Traditional Approach

Openings that are circular, square, or nearly square in shape may be considered as large openings provided that the depth (or diameter) of the opening is in a realistic proportion to the beam size, say, about greater than 0.40 times the overall beam depth. The introduction of a large opening in a reinforced concrete beam would normally reduce its load-carrying capacity considerably. However, it is possible to reinforce such a beam, restoring its strength to that of a similar solid beam.

2.2.4.1 Available Design Procedures

In the next proposed design procedure, the ACI code [3] has been followed throughout unless otherwise stated. In general, the design of reinforced concrete structures involves:

- **Structural analysis**, whereby the structure is analyzed to determine the distribution of shear forces and moments due to ultimate loads. All possible loading combinations are considered, and the bending moment and shear forces envelopes are determined accordingly.
- **Strength design**, wherein the critical sections are designed for ultimate strength in bending, shear, and torsion. The strength requirements are fulfilled throughout the whole structure.
- **Serviceability design,** to ensure that the structure performs its intended functions satisfactorily under working loads.

Structural Analysis

In the case of a statically determinate beam, the shear force and bending moment envelopes can be obtained from statics.

Bending moment and shear force envelopes

The beam can be analyzed for all possible load combinations by any finite element elastic method to obtain the shear force and bending moment envelopes.

Design for Strength

Knowing the bending moment and shear force envelopes, the solid segments of the beam can be designed in the usual manner. The recommended design process for the opening segment is based on the observed Vierendeel truss behavior of chord members at an opening. That is, consistent with test results, contraflexure points are assumed at midspan of chord members, for which the axial load is obtained by dividing the beam moment at the center of the opening by the distance between the plastic centroids of the chord members. The shear force acting at the center of the opening is distributed between the chord members according to their relative flexural stiffnesses. Such an assumption has been found to give a realistic distribution of the applied shear (Barney et al., 1977) and simplifies the calculation. Methods used in the design; plastic hinge method and strut-and-tie method. The steps involved are summarized as follows:

Step 1: Forces and moments in chord members

In practice, beams are usually subject to combined bending and shear. Figure 5.12 shows a simply-supported reinforced concrete beam with an opening, subjected to a uniform load. The free-body diagram at the beam opening can be represented as in Fig. 5.12b, and the free-body diagrams of the chord members above and below the opening as in Fig. 5.12c. It is observed that the unknown action effects at the center of the opening are the axial forces (N_t and N_b), the bending moments (M_t and M_b), and the shear forces (V_t and V_b) in the chord members. There are three equilibrium equations relating these six unknowns. These are:

$$M_t + M_b + N_z = M_m \quad (5.16)$$

$$N_t + N_b = 0 \quad (5.17)$$

$$V_t + V_b = V_m \quad (5.18)$$

In which M_m and V_m are the applied moment and shear force, respectively, at the center of the opening. Determine the ultimate design bending moment, M_m, and shear force, V_m, at the middle of the opening segment from bending moment and shear force envelopes obtained from the global action, and calculate axial forces N_t and N_b (positive for compression) acting in the top and bottom chords, respectively, as:

$$N_t = \frac{M_m}{Z} \quad (5.19)$$

$$N_b = -N_t \quad (5.20)$$

Where Z is the distance between the plastic centroids of the top and bottom chords. Distribute the applied shear between the top and bottom chords in proportion to their gross flexural stiffnesses as:

$$V_t = V_m \left(\frac{I_{gt}}{I_{gt}+I_{gb}}\right) \quad (5.21)$$

$$V_b = V_m \left(\frac{I_{gb}}{I_{gt}+I_{gb}}\right) \quad (5.22)$$

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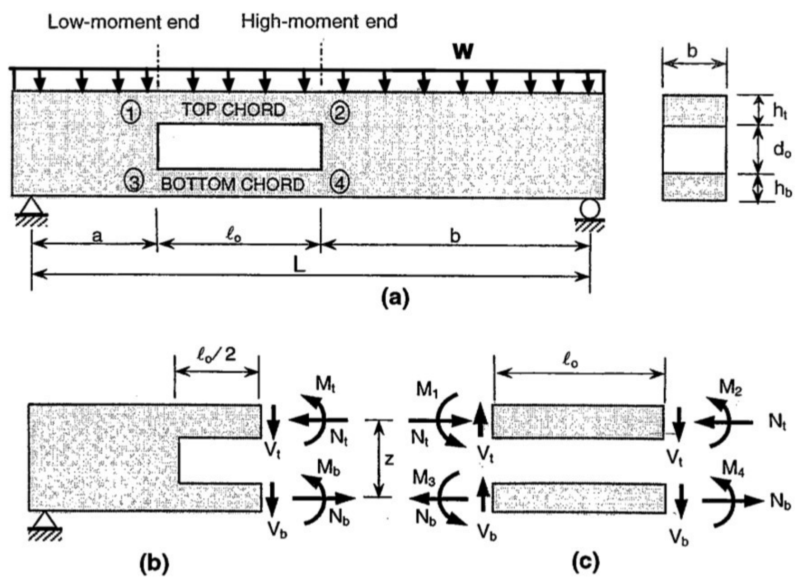


Figure 5.12 Beam with an opening under bending and shear. (a) The beam; (b) Free-body diagram of opening segment; (c) Free-body diagram of the chords.

Where V_t and V_b are the shear forces carried by the top and bottom chords, respectively.

Calculate moments at the ends of the top and bottom chords from statics (refer to Fig. 5.12):

$$M_1 = -\frac{W\ell_o^2}{8} - \frac{V_t\ell_o}{2} \tag{5.23}$$

$$M_2 = -\frac{W\ell_o^2}{8} + \frac{V_t\ell_o}{2} \tag{5.24}$$

$$M_3 = -\frac{V_b \ell_o}{2} \tag{5.25}$$

$$M_4 = \frac{V_b \ell_o}{2} \tag{5.26}$$

Where W is the uniformly distributed load acting directly on top chord and M is the moment. The subscripts 1, 2, 3, and 4 designate the opening corners as shown in Fig. 5.12. The collapse mechanism consistent with experimental observations, the assumed mechanism consists of four hinges in the chord members, with one at each corner of the opening, as shown in Fig. 5.13

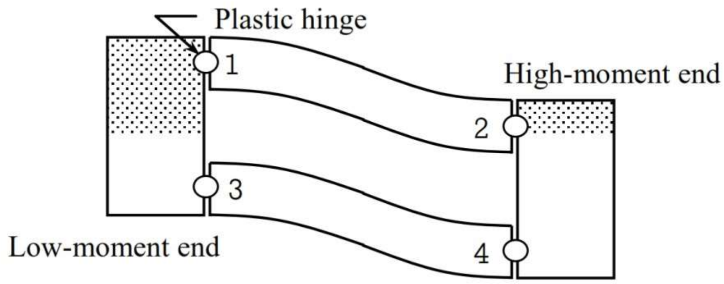


Figure 5.13 Assumed collapse mechanism for a beam with large openings.

When the chord members are symmetrically reinforced then the moments at two ends of each chord member (potential hinge location) are numerically the same at plastic collapse. That is, $M_1 = M_2$ and $M_3 = M_4$. From the free-body diagram of the chord members (Fig. 5.12c), it may be readily shown that the contraflexure points occur at midpoint of the chord members. This means that $M_t = 0$ and $M_b = 0$.

Step 2: Stability of compression chord

When the section being analyzed is a T-beam, the effective width of flange in determining the properties and capacities of the compressive strut should not exceed the limits set by the ACI Code [3]. Where the opening segment is subject to positive bending (for example, in the mid span region of a continuous beam), the compression (top) chord will be restrained by the continuity of the slab and, thus, may be considered as a non-sway frame member for which, according to ACI Code, the effect of slenderness may be neglected when:

$$\frac{K\ell_u}{r} < 34 - 12\frac{M_{1b}}{M_{2b}} \tag{5.27}$$

in which the effective length factor K is taken as 1, ℓ_u is the unsupported length of the compression chord, and r is the radius of gyration. The value of M_{1b} and M_{2b}can be taken as M_3 and M_1, respectively, with the signs as directed by ACI Code. According to the Code, r can be taken as:

$$r = 0.3d_c \tag{5.28}$$

where d_c can be taken as the depth of the compression chord. However, when an opening segment is subject to negative bending (for example, in between the inflection points and the support of a continuous beam), the compression (bottom) chord should be considered as a non-sway frame member for which, according to ACI Code, the effects of slenderness may be neglected when:

$$\frac{K\ell_u}{r} < 22 \tag{5.29}$$

If Eq. (5.13) or Eq. (5.15) is not satisfied, the moment magnification method as described in ACI Code may be used to design the compression chord. However, it is suggested that the dimensions of the compression chord be revised so as to eliminate the effects of slenderness.

Step 3: Design of longitudinal reinforcement for chord members

The longitudinal reinforcement in the top and bottom of the solid section adjacent to the opening should be continued throughout the opening segments. Additional reinforcement required to resist the combined moment and axial force in each chord member is designed and, as a trial, it could be such that each chord is symmetrically reinforced. Use the same amount and arrangement at its bottom as additional reinforcement required to restore the strength and avoid brittle failure of the beam due to the provision of openings. With the reinforcement for the chord members so decided, the corresponding idealized column interaction diagrams can be constructed by the method of strain compatibility. The critical combinations of bending moment and axial load for the chord members as determined earlier are then plotted in the interaction diagrams. If all the combinations fall within the appropriate interaction diagrams, the reinforcement provided will be sufficient.
Otherwise, a revision of reinforcement is necessary. Also, the flexural capacity of the top chord should be sufficient to support any direct external loading.

Step 4: Design of shear reinforcement for chord members

The shear forces carried by the top and bottom chords are given by Eqs. (5.7a) and (5.7b), respectively. Knowing these forces, the required amount of reinforcement can be designed in a manner similar to reinforced concrete beams and slabs. However, according to ACI Code, the effects of axial forces in the chord members must be accounted for in design. For a T-beam where the opening is placed flushed with the flange, the top chord can be considered as a slab. Although the flange may be too shallow for effective placement of shear reinforcement, the shear stresses are usually low and, consequently, shear reinforcement would not usually be necessary in the top chord.

Step 5: Multiple openings and design of post between openings

When multiple openings are placed close to each other in a beam, the element between two adjacent openings is known as a post. Proper design and detailing for the post should be provided. Tests carried out at the laboratory of Portland Cement Association (ACI-ASCE, 1973) have indicated that closely spaced multiple openings can be placed in a beam if each opening has adequate side reinforcement. Specimens with multiple circular and oval holes failed in the chord members when the width of the post was equal to or greater than 3/8 the depth of the web. Figure 5.14 shows an inverted T-beam with multiple rectangular openings separated by adequately reinforced posts after it has been tested to failure.

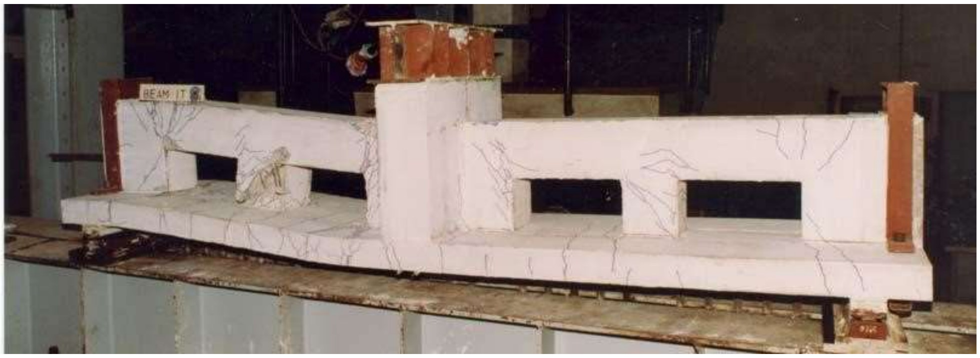


Figure 5.14 Failure of a beam with multiple rectangular openings separated by adequately reinforced post.

To ensure that the posts behave rigidly, Barney et al. (1977) recommended that adjacent openings should be separated by posts having overall width-to-height ratios of at least 2.0 where the width of the posts is the distance between adjacent stirrups. It was also suggested that nominal design shear stress for the posts be limited to $0.17\sqrt{f_c'}$ (MPa).

When two openings are placed close to each other, it is evident from the free-body diagram shown in Fig. 5.15 that a horizontal shear, V_p, compression force, N_p, and bending moment, M_p, act on the post between the openings. Assuming that points of contraflexure occur at the midlength of the chord members in each opening, equilibrium of forces gives:

$$V_p = T_2 - T_1 \tag{5.30}$$

$$N_p = V_{b1} - V_{b2} \tag{5.31}$$

$$M_p = (T_2 - T_1)\left(d_o + \frac{d_b}{2}\right) - V_{b1}\left(\frac{\ell_{o1} + W_p}{2}\right) - V_{b2}\left(\frac{\ell_{o2} + W_p}{2}\right) \tag{5.32}$$

Where W_p = width of post, taken as the distance between vertical stirrups in post adjacent to the sides of the two openings; T = tensile force acting on the bottom chord member of opening; V_b = vertical shear force acting on the bottom chord member of opening; e = eccentricity of opening; d_p = depth of bottom chord member; ℓ_o = length of opening, taken as the distance between the vertical stirrups adjacent to the two sides of the opening; do = depth of opening; and subscripts 1 and 2 refer to the openings to the left and right of post, respectively. Knowing the values of V_p, N_p, and M_p, the required reinforcement can be obtained by designing the post as a short, braced column.

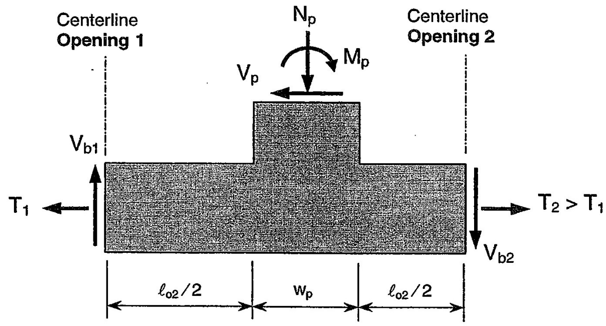


Figure 5.15 Forces acting on the post between adjacent openings.

Step 6: Design for serviceability

The two important serviceability requirements to be met are cracking and deflection.

a- Cracking

Assuming that the crack control requirements of the solid segments are met either by proper reinforcement detailing or by physical calculation, the following crack control provisions are recommended for the critical sections at corners of the opening. At each vertical edge of the opening, a combination of vertical stirrups and diagonal bars would be used with a shear concentration factor, □, of 2 such that at least 50% of the shear resistance is provided by the diagonal bars (Tan, 1982). Thus, for each side of the opening, the required area of vertical stirrups, A_v, is given by:

$$A_v = \frac{0.50\ (\eta\ V)}{\phi\ f_{yv}} \tag{5.33}$$

In which V, ϕ, and f_{yv} are the design shear, capacity reduction factor, and yield stress of stirrups, respectively. The vertical stirrups should be placed as close to the edge of the opening as permitted by the required concrete cover. The required area of diagonal reinforcement, A_d, is given as

$$A_d = \frac{0.50\ (\eta\ V)}{\phi\ f_{yd} \sin\alpha} \tag{5.34}$$

Where f_{yd} is the yield stress and α is the angle of inclination of the diagonal bars to the beam axis. To avoid confusion during construction and to account for any possible load reversal, the same amount of diagonal reinforcement should be provided both at the top and bottom corners of the opening.

b- Deflection

The indirect way of satisfying the serviceability requirement of deflection by limiting the spaneffective depth ratio is not valid for a beam with openings. Therefore, an estimate of the actual service load deflection is necessary. For this purpose, the method used for the analysis of the beam at ultimate load may be used. Since the reinforcement details are fully known, a conservative estimate of service load deflection may be obtained and checked against code requirements by using the cracked moment of inertia of various segments.

The model shown in Fig. 5.16 considers that the chord members act as struts framing into rigid abutments on each side of the opening. The effective length, ℓ_e, of the struts is conservatively taken as the distance between the full-depth stirrups on each side of the opening. To reflect the Vierendeel truss action observed in the tests, points of contraflexure are assumed at the mid-length of each strut. Thus each half of the chords bends as a cantilever, as shown. Denoting the moments of inertia for the top and bottom struts as I_t and I_b, respectively, the relative displacement of one end of the opening with respect to the other under the action of V may be obtained as

$$\delta_v = \frac{V\ell_e^3}{12\ E_c(I_t+I_b)} \tag{5.35}$$

Where E_c is the modules of elasticity of concrete. Under service load, I_t may be based on gross concrete section while I_b can be conservatively based on a fully cracked section.

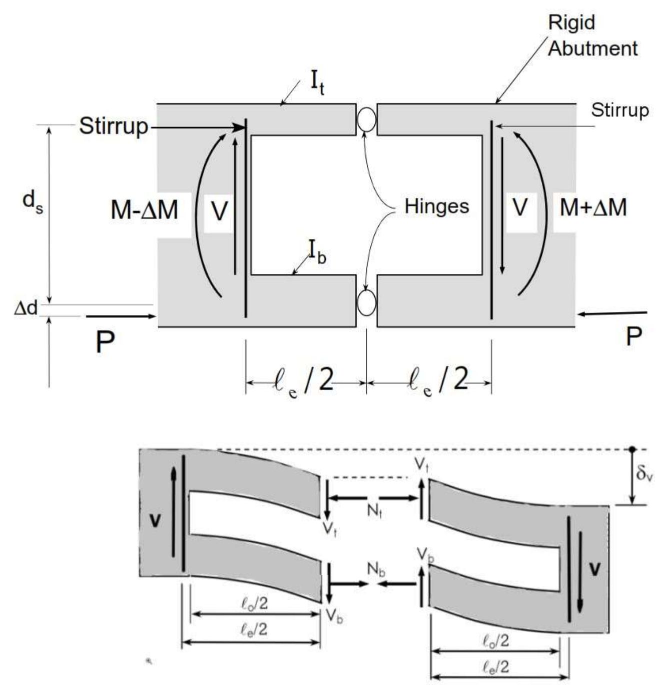


Figure 5.16 Idealized model for the estimation of deflection at opening (Barney et al., 1977).

The maximum deflection of the beam can be calculated as

$$\delta = \delta_w + \delta_v \tag{5.36}$$

Where $_w$ is the maximum deflection in the absence of opening. A more rigorous method to calculate deflections that entails an elastic analysis is also available (Mansur et al., 1992). In the method, the beam is treated as a structural member with several segments constituting the portions with solid beam sections and those with sections traversed by the opening. An equivalent stiffness is adopted for the latter segments and the beam can be analyzed using methods such as the Direct Stiffness Method to obtain the maximum beam deflection under service load.

Step 7: REINFORCEMNT DETAILING

The reinforcement details for the solid segments of the beam should follow the normal detailing procedure for simple and continuous beams. The opening segment requires additional reinforcement, and it should be detailed carefully keeping in mind the strength and crack control requirements.

Similar to a beam with small openings, incorporation of a large opening in the pure bending zone of a beam will not affect its moment capacity provided that the depth of the compression chord h_c is greater than or equal to the depth of ultimate compressive stress block a, and that instability failure of the compression chord is prevented by limiting the length of the opening (Mansur and Tan, 1999).

In practice, openings are located near the supports where shear is predominant. In such a case, tests have shown that a beam with insufficient reinforcement and improper detailing around the opening region fails prematurely in a brittle manner (Siao and Yap, 1990). When a suitable scheme consisting of **additional longitudinal bars** near the top and bottom faces of the bottom and top chords, to resist the combined moment and axial force in each chord member is designed and, as a trial, it could be such that each chord is symmetrically reinforced, as shown in Fig. 5.17.

Short stirrups in both the chords, as shown in Fig. 5.17, to resist the shear forces carried by the top and bottom chords, then the chord members behave in a manner similar to a Vierendeel panel and failure occurs in a ductile manner. The failure of such a beam is shown in Fig. 5.18. Clearly, the failure mechanism consists of four hinges, one at each end of the top and bottom chords.

The critical section for cracking at corners of the opening, at each vertical edge of the opening, a combination of **vertical stirrups** and **diagonal bars** would be used. At least 50% of the shear resistance is provided by the diagonal bars (Tan, 1982).

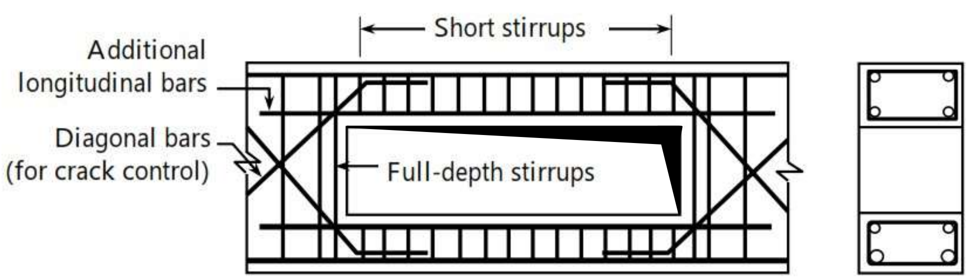


Figure 5.17 A suitable reinforcement scheme for the large opening.

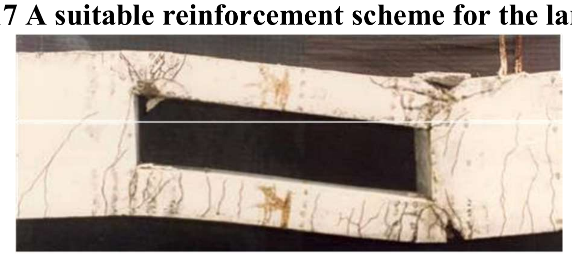


Figure 5.18 Ductile failure of a beam under combined bending and shear.

2.2.4.2 Numerical Example Case 2- Beam (Group C) with Large Rectangular Openings 100×300mm

A simply supported rectangular beam 100mm wide and 250mm height carries two concentrated and symmetrical factored loads 20.5kN on a span of 2.0m is shown in Fig. 5.19. The beam having a rectangular openings 100×300mm, located at the shear-span of the beam. Provide a suitable design for the beam with particular emphasis on the opening segment of the beam. Given $f_c' =$, $f_y =$, and $f_{yv} =$

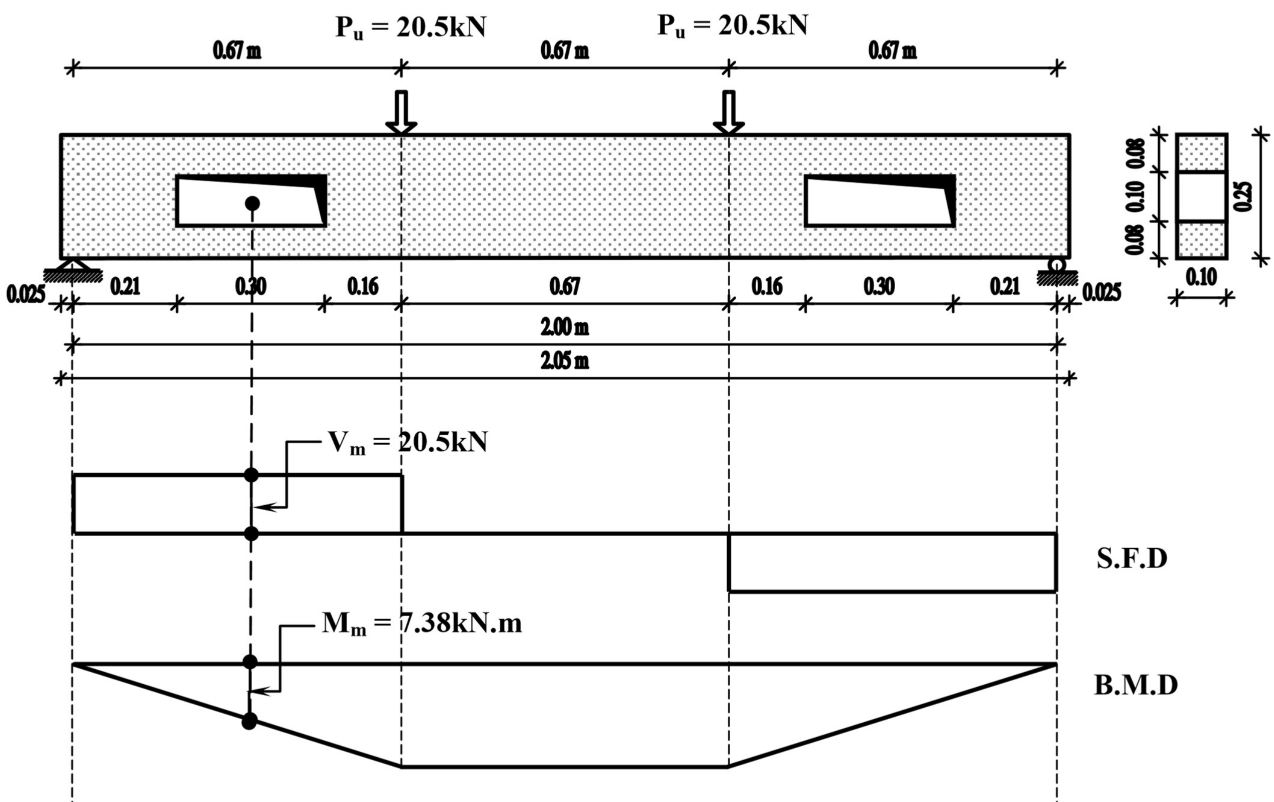


Figure 5.19 Beam and loading.

Solution:

1. Structural Analysis

(a) Bending moment and shearing force diagrams

Bending moment and shearing force diagrams for this beam are shown in Fig. 5.19.

2. Design for Strength

(a) Solid section

The flexural design of the solid section at mid-span of the beam is as follows:

$$M_u = 0.85 f_c' ba \times \left[d - \frac{a}{2}\right]$$

$$13.74 \times 10^6 = 0.85 \times 52 \times 100 \times a \times \left[210 - \frac{a}{2}\right]$$

$$\text{We get } a = 15.36\text{mm}$$

$$\mathrm{T} = C$$

$$A_s f_y = 0.85 f_c' ba$$

$$A_s \times 400 = 0.85 \times 52 \times 100 \times 15.36$$

$$A_s = 169.728\text{mm}^2 \rightarrow \text{Use } 3\Phi 10$$

(b) Opening segment

For the opening segment, the axial loads and shear forces in chord members (refer to Fig. 5.12) are evaluated from the bending moment M_m and shear force V_m at the center of opening using Eqs. (5.19) to (5.22) with $\mathrm{Z} = 250 - 80 = 170\text{mm}$.

$$N_t = \frac{\mathrm{M_m}}{\mathrm{Z}} = \frac{7.38}{0.17} = 43.41kN \qquad (Compression)$$

$$N_b = -N_t = -43.41kN \qquad (Tension)$$

The secondary moments of the critical end sections, calculated by Eqs. 5.23 to 5.26 are as follows:

$$\mathrm{I_{gt}} = \mathrm{I_{gb}} = \frac{100 \times 80^3}{12} = 4.27 \times 10^6 mm^4$$

and

$$\mathrm{V_t} = \mathrm{V_m}\left(\frac{\mathrm{I_{gt}}}{\mathrm{I_{gt}} + \mathrm{I_{gb}}}\right) = 20.5\left(\frac{4.27 \times 10^6}{4.27 \times 10^6 + 4.27 \times 10^6}\right) = 10.25kN$$

$$\mathrm{V_t} = \mathrm{V_b} = 10.25kN$$

$$M_1 = -\frac{W\ell_o^2}{8} - \frac{V_t \ell_o}{2} = 0 - \frac{10.25 \times 0.3}{2} = -1.54kN.m$$

$$M_2 = -\frac{W\ell_o^2}{8} + \frac{V_t\ell_o}{2} = 0 + \frac{10.25 \times 0.3}{2} = +1.54kN.m$$

$$M_3 = -\frac{V_b\ell_o}{2} = -\frac{10.25 \times 0.3}{2} = -1.54kN.m$$

$$M_4 = \frac{V_b\ell_o}{2} = \frac{10.25 \times 0.3}{2} = +1.54kN.m$$

Since the top chord of the opening segment is under compression, it may be considered a nonsway frame member for which, according to ACI Code, the effect of slenderness may be neglected when

$$\frac{K\ell_u}{r} < 34 - 12\frac{M_{1b}}{M_{2b}}$$

Since $r = 0.3d_c = 0.3 \times 80 = 24mm \text{ and } \ell_u = 300mm,$

$$\frac{K\ell_u}{r} = \frac{1\times300}{24} < 34 - 12\frac{1.54}{1.54} < 22$$

Hence, the compression chord is satisfactory with regarding stability.

Longitudinal reinforcement: The solid section adjacent to the opening has $3\Phi10$ bottom and $2\Phi10$ top rebars. Because Contraflexure points occur at midspan, symmetrical arrangements of reinforcement to be provided to top of the bottom chord ($3\Phi10$) and $2\Phi10$ to the bottom of the top chord. This is the amount of reinforcement to be checked using the interaction diagrams for the chord members. If all points fall within the respective interaction diagrams, the amount of reinforcement provided is satisfactory.

Shear reinforcement: The shear at opening center is 20.5kN. Therefore, the design shear force is

$$V_t = V_b = 10.25kN$$

The top chord is subject to combined bending and axial compression, thus the shear strength of concrete is (according to the ACI Code [3], sec. 11.2.1.2) as follows:

$$V_c = \phi \times \frac{1}{6}\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f_c'}b_w\, d \qquad (5.37)$$

But not less than zero, where λ = for s nd-lightweight concrete and 0.75 for all-lightweight concrete, N_u is positive for compression, and N_u/A_g shall be expressed in MPa.

$$V_c = 0.85 \times \frac{1}{6}\left(1 + \frac{43.41 \times 10^3}{14 \times 80 \times 100}\right) \times 1 \times \sqrt{52} \times 100 \times 60 = 8.51kN < 10.25kN$$

The top chord is treated as a beam, therefore a minimum amount of links must be provided. The maximum spacing limit is $d/2 = 60/2 = 30mm$. Therefore, provide stirrups of diameter 8mm spaced at 30mm.

The bottom chord is subject to combined bending and axial tension, thus the shear strength of concrete is (according to the ACI Code [3], sec. 11.2.2.3) as follows:

$$V_c = \phi \times \frac{1}{6}\left(1 + \frac{0.29N_u}{A_g}\right)\lambda\sqrt{f_c'}b_w\, d \qquad (5.38)$$

Where N_u is negative for tension and N_u/A_g shall be expressed in MPa.

$$V_c = 0.85 \times \frac{1}{6}\left(1 + 0.29 \times \frac{-43.41 \times 10^3}{80 \times 100}\right) \times \sqrt{52} \times 100 \times 60 = 3.52kN < 10.25kN$$

2. Design for Serviceability

(a) Cracking

Assuming that the crack control requirements of the solid segments are met either by proper reinforcement detailing or by physical calculation. For the opening segment, the maximum shear is $V_m = 20.5$kN.

Assuming □ = 2, α = 45^0, $f_{yv} = 240$, and $f_{yd} = 450$, and 75% of total shear is carried by the diagonal bars, the required areas of vertical stirrups and diagonal bars (refer to Eqs. 5.33 and 5.34) are:

$$A_v = \frac{0.25\,(\eta\, V)}{\phi\, f_{yv}} = \frac{0.25 \times 2 \times 20.5}{0.85 \times 240} = 0.05m^2 = 50.25mm^2 \rightarrow \text{Use } 1\emptyset 6$$

$$A_d = \frac{0.75\,(\eta\, V)}{\phi\, f_{yd}\, sin\, \alpha} = \frac{0.75 \times 2 \times 20.5}{0.85 \times 450 \times sin\, 45^0} = 0.1137m^2 = 113.7mm^2$$

Use $2\Phi10$ ($157mm^2$) diagonal bars at each corner of the opening.

(b) Deflections

The service load is $P_s = P_u/1.7 = 20.50/1.7 = 12.1kN$, and the corresponding shear force is 12.1kN. The effective length of chord members for deflection calculations is taken as

515

$$\ell_e = 300 + 50 = 350\text{mm}$$

Based on gross section properties, the moment of inertia of the chord members is equal to $100 \times 80^3/12$ or $4.27 \times 10^6\text{mm}^4$. For the estimation of deflection, conservatively assume

$$I_t = 4.27 \times 10^6\text{mm}^4;\ I_b = 0.1 \times I_t = 4.27 \times 10^5\text{mm}^4$$

Also, the modulus of elasticity of concrete is

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{52} = 34 \times 10^3\text{MPa}$$

Hence, from Eq. (5.35), the deflection due to shear force at the opening is

$$\delta_v = \frac{V\ell_e^3}{12\ E_c(I_t+I_b)} = \frac{12.1\times10^3\times350^3}{12\times34\times10^3(4.27\times10^6+4.27\times10^5)} = 0.27\text{mm}$$

For the load arrangement shown in Fig. 5.19, the midspan deflection for a beam without openings, ${}_{w}$, is

$${}_{w} = \frac{23}{648}\frac{PL^3}{E_c I} = \frac{23}{648} \times \frac{12.1\times10^3 \times 2000^3}{34\times10^3\times130.21\times10^6} = 0.78\text{mm}$$

Hence, the total midspan deflection of the beam can be calculated as

$$\delta = \delta_v + \delta_w = 0.27 + 0.78 = 1.05mm < L/360 = 5.6mm$$

3. Details of Reinforcement

The reinforcement details for the solid segments of the beam should follow the normal detailing procedure for simple beams. The opening segments require additional reinforcement, and it should be detailed carefully keeping in mind the strength and crack control requirements. Figure 5.20 shows the final arrangement of reinforcement for the opening segment.

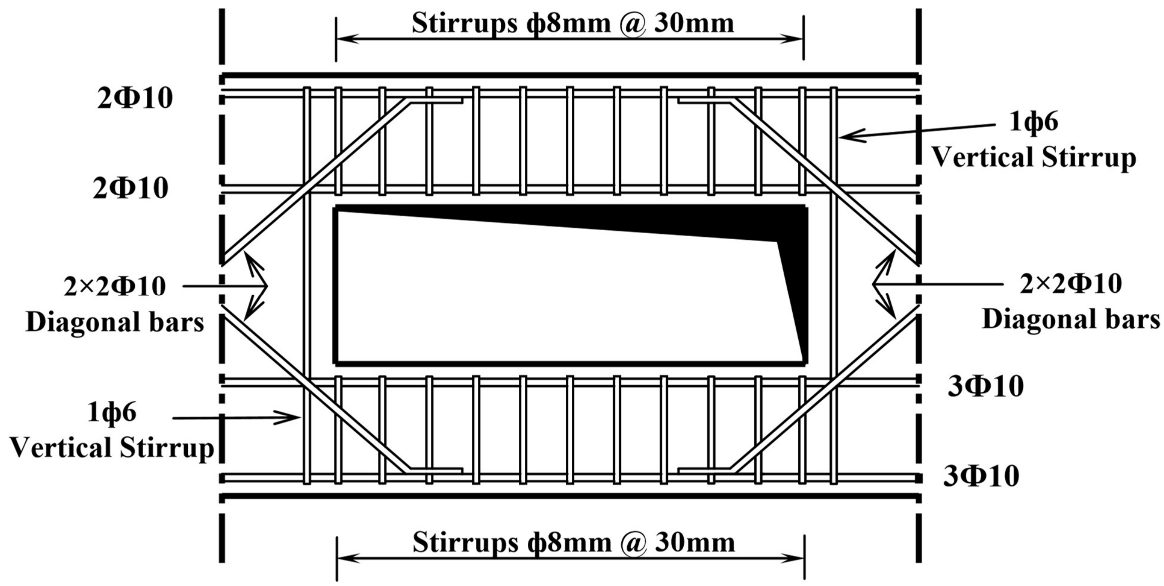


Figure 5.20 Reinforcement details at opening segment.

2.2.5 Redesign for the previous Case 2-Beam (Group C) with Large Rectangular Opening 100×300mm using Strut-and-Tie Method

The first step in this method is to visualize the flow of forces from the applied loads to the supports. This is accomplished from the obtained elastic principal stress trajectories from the 3D nonlinear finite element analysis. Compression stress trajectories will be replaced by compression elements (Struts) and tension stress trajectories will be replaced by tension elements (Ties). Once the model is established, the forces in the struts and ties can be calculated from statics. The required reinforcements of the tension ties can then be chosen.

(a) Developing the strut-and-tie model

A strut-and-tie model for the considered beam is shown in Fig. 5.21. Here, the compressive struts are shown in dotted lines while tension ties are shown in solid lines.

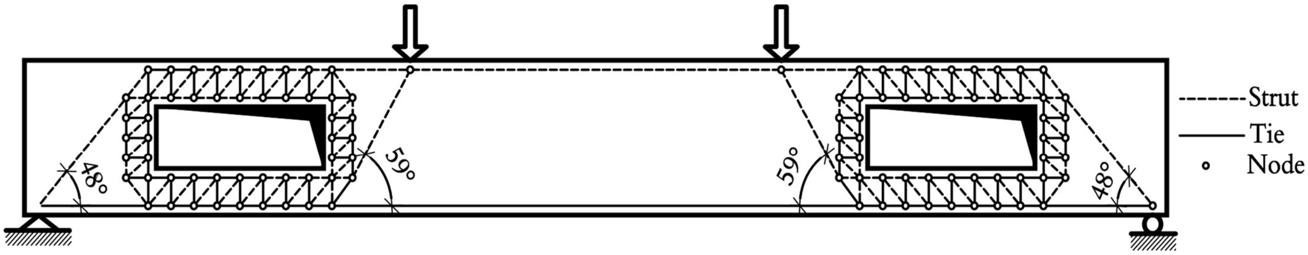
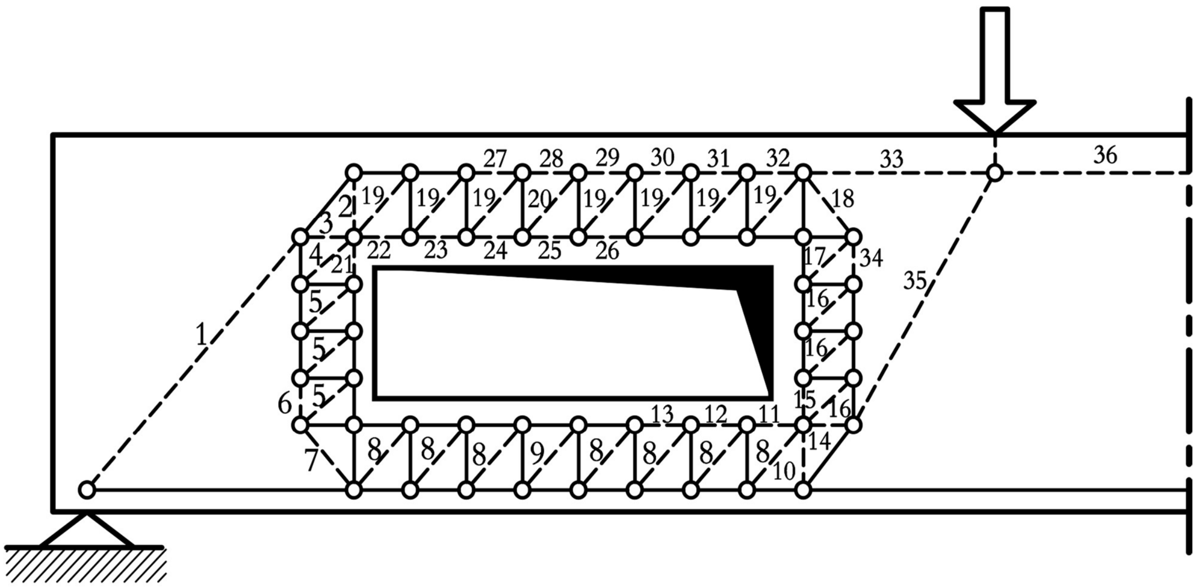
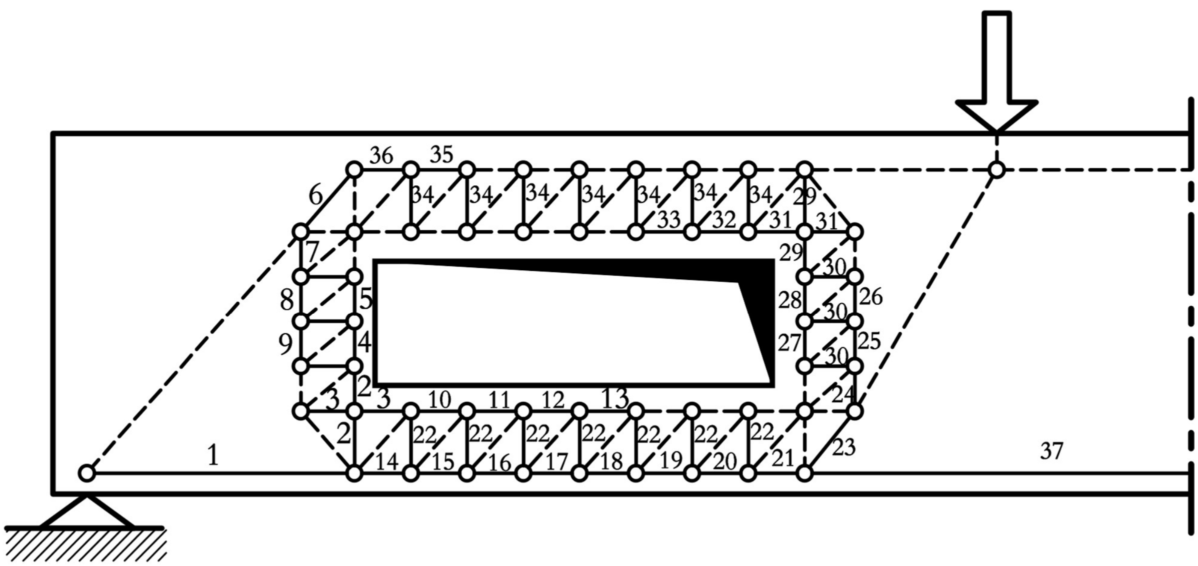


Figure 5.21 Proposed strut-and-tie model.



(a) Strut labels for strut-and-tie model.



(b) Tie labels for strut-and-tie model.

Figure 5.22 Details of the strut-and-tie model.

(b) STM forces

The forces in all members are determined from statics and their magnitudes in kN are as indicated in Table 5.2. The struts, ties, and nodes are labeled as in Fig. 5.22.

Table 5.2 STM forces.

Model Label	Force, kN	T or C	Model Label	Force, kN	T or C	Model Label	Force, kN	T or C	Model Label	Force, kN	T or C
1	25.97	C	20	15.72	C	3	39.70	T	22	10.68	T
2	17.35	C	21	7.74	C	4	21.42	T	23	8.95	T
3	33.30	C	22	42.91	C	5	6.84	T	24	27.49	T
4	27.49	C	23	33.09	C	6	24.54	T	25	17.65	T
5	24.30	C	24	23.27	C	7	37.85	T	26	7.80	T
6	10.74	C	25	10.99	C	8	18.41	T	27	2.02	T
7	32.42	C	26	1.17	C	9	3.84	T	82	11.87	T
8	13.67	C	27	2.29	C	10	31.15	T	82	25.00	T
9	15.10	C	28	12.12	C	11	22.61	T	03	13.13	T
10	6.99	C	29	24.39	C	12	14.07	T	03	28.30	T
11	22.24	C	30	34.22	C	13	3.39	T	08	18.48	T
12	13.70	C	31	44.04	C	14	4.23	T	00	8.66	T
13	5.15	C	32	53.86	C	15	12.77	T	03	9.82	T
14	17.65	C	33	48.51	C	16	21.32	T	03	7.53	T
15	7.82	C	34	2.05	C	17	31.99	T	03	17.35	T
16	16.41	C	35	23.78	C	18	40.54	T	03	60.57	T
17	18.57	C	03	60.57	C	19	49.08	T	-	-	-
18	21.46	C	1	15.94	T	20	57.62	T	-	-	-
19	13.89	C	2	36.00	T	21	66.16	T	-	-	-

T = Tension (Tie) and C = Compression (Strut)

(c) Rebar's Sizes

Longitudinal reinforcement

Bottom chord member

For the top reinforcement, the maximum tensile force is 39.70kN. Therefore, the required area is

$$39.70 \times 10^3/450 = 88.22mm^2$$

Therefore, provide $2\Phi 8$ bars at the top of the bottom chord.

For the bottom reinforcement, the maximum tensile force is 60.57kN. Hence, the required area is

$$60.57 \times 10^3/450 = 134.60mm^2$$

Therefore, provide $2\Phi 10$ bars at the top of the bottom chord.

Top chord member

The maximum compressive force in strut S_{36} is 60.57kN. Referring to the prismatic section on the right of the beam under the same moment and shear force, the concrete may be assumed to take 67.90kN, no remaining force to be resisted by compressive steel reinforcement. So concrete carried all compressive force, so use compression reinforcement of $2\phi 6$ (as a minimum) only at the top of the top chord member to anchor vertical stirrups. The maximum tensile force is 28.30kN. Hence, the required area of bottom steel is

$$28.30 \times 10^3/450 = 62.90mm^2$$

Therefore, provide $2\Phi 8$ bars at the bottom of the top chord.

Transverse reinforcement

Top and bottom chords

The maximum tensile force in the vertical Tie T_{22} is 10.68kN. Therefore,

$$T_{22} = 2leg \times A_{sti.}f_y$$

$$T_{22} = 2leg \times \left(\frac{\pi\phi_{sti.}^2}{4}\right) f_y$$

$$10.68 \times 10^3 = 2leg \times \left(\frac{\pi\phi_{sti.}^2}{4}\right) \times 240$$

Therefore, provide $\phi 6\text{mm}$ stirrups (2-legs) spaced at 40mm. Sides of opening

Left side

The maximum tensile vertical force in Tie T_2 is 36kN. Therefore, the area of shear reinforcement required at the low-moment end of the left side of opening is

$$36 \times 10^3/240 = 150\text{mm}^2$$

Therefore, provide $2\phi 8\text{mm}$ stirrups (2-legs full depth, 201.06mm^2) to the left of opening. Also use $2\phi 8\text{mm}$ stirrups (2-legs full depth) to resist the force in tie T_7 which is 37.85kN.

Right side

The maximum tensile vertical force in Tie T_{29} is 25kN. Therefore, the area of shear reinforcement required at the high-moment end of the right side of opening is

$$25 \times 10^3/240 = 104.17\text{mm}^2$$

Therefore, provide $2\phi 6\text{mm}$ stirrups (2-legs full depth, 113.10mm^2) to the right of opening. Also use $2\phi 6\text{mm}$ stirrups (2-legs full depth) to resist the force in Tie T_{24} which is 27.49kN.

(d) Design for crack control

In this example, use diagonal reinforcement to avoid cracking only under service load conditions. The factored shear at the center of opening is $V_u/ = 24.12\text{kN}$ and the design shear (assuming a shear concentration factor of 2) is 2×24.12 or 48.24kN. Therefore, provide additional diagonal bars at 45^0 to resist the remaining shear. The required area is

$$A_d = \frac{(48.24 - 24.12) \times 10^3}{(450 \times \sin 45^0)} = 75.80\text{mm}^2$$

Use $2\Phi 8$ (100.53mm^2) diagonal bars at each corner of the opening.

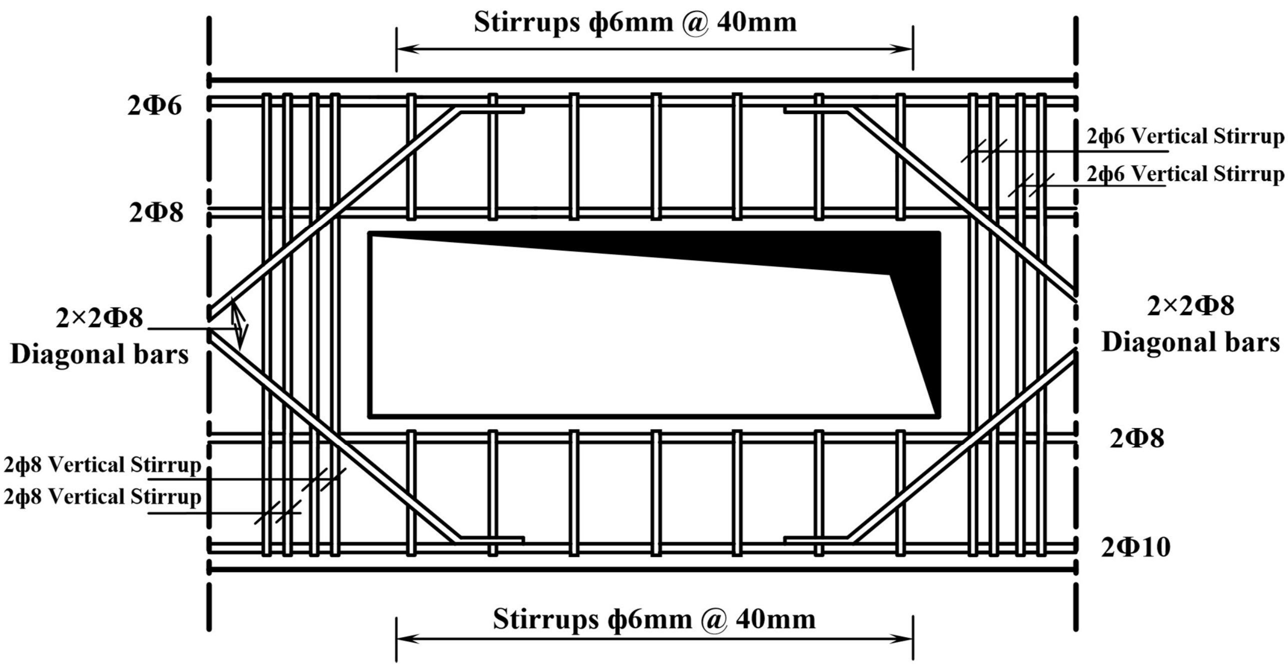


Figure 5.23 Reinforcement details at opening segment using STM.

3.3 DEEP BEAMS

Some of the existing design codes; e.g., the ACI 318M-11 Code [3] and the Egyptian Code (2007) [10], define a beam to be **deep** when:

1. ACI 318 M-11 Codes [3]:
   - ℓ_n /h ≤ 4 or
   - a/h ≤ 2.

2. Egyptian Code (2007) [10]:
   - L/d ≤ 4 or
   - a/d ≤ 2.

3.3.1 A general Procedure for Strut-and-Tie Modeling for Discontinuity Regions

The process used in the developing STM models for discontinuity regions is illustrated in Figure 5.24.

STEP 1 – DISTINGUISH THE D-REGIONS

As discussed in previous, using St Ven nt's principle,   D-region is assumed to extend a distance equal to the largest cross-sectional dimension of the member away from a geometrical discontinuity or a large concentrated load. The determined B-/D-region interface is the assumed location where the stress distribution becomes linear again. Using this basic assumption, the Dregions can be described. The entire deep beam usually is a disturbed region.

STEP 2 – DETERMINE THE BOUNDARY CONDITIONS OF THE D-REGIONS
Once the extent of a D-region has been determined, the bending moments, shear forces, and axial forces must be determined at the B-/D-region interface from analysis of the B-region and are then used to determine the stress distributions at the B-/D-region interface. These stress distributions can then be modeled as equivalent point loads having locations and magnitudes which can be determined directly from the stress distributions. When determining the boundary conditions on the B-/D-region interface, it is essential that equilibrium be maintained on the boundary between B-and D-regions. If the bulk of the structure falls into a D-region it may be expedite to use a global model of the structure and use the external loads and reactions as the boundary conditions.

Figure 5.24 Flowchart illustrating STM steps, Brown et al. [9].

STEP 3 – SKETCH THE FLOW OF FORCES
After the stress distributions acting on the B-region/D-region interface have been modeled as equivalent point loads, the flow of forces through the D-region should be determined. For most design cases, the flow of forces can easily be seen and sketched by the designer. When the flow of forces becomes too complex to be approximated with a sketch, a finite element analysis can be used to determine the flow of forces through a reinforced concrete structural member. For most D-regions, such efforts are unwarranted since the stress paths can be estimated easily. Another method used to determine the flow of forces is the load path method as proposed by Schlaich et al. (1987).

STEP 4 – DEVELOP A STM
A STM should be developed to model the flow of forces through the D-region determined in the previous step. When developing a STM, try to develop a model that follows the most direct force path through the D-region. Also, avoid orienting struts at small angles when connected to ties. According to Collins and Mitchell, as the angle between a strut and tie decreases, the capacity of the strut also decreases (1986). For this reason, many design specifications specify a minimum angle between struts and ties of about 25 degrees. It should be noted that the AASHTO LRFD provisions do not specify a minimum angle between struts and ties; however, the limiting strut compressive stress equation defined in the specification is a function of the angle between the strut and tie and decreases as the angle between the strut and tie decreases.

A D-region may be subjected to more than one type of loading. It is imperative that a STM be developed and analyzed for each different loading case. On a similar note, for a given load case for a D-region, more than one STM can be developed. Schlaich and schäfer [37] suggested that models with the least and shortest ties are the best. In addition, Schlaich and schäfer also suggested that two simple models can sometimes be superimposed to develop a more sophisticated model that better models the flow of forces through a D-region.

Also, Brown et al. [9], expl ined th t "it is prefer ble to h ve model th t is st tic lly determin nt " St tic lly determin nt models require no knowledge of the member stiffnesses which makes it simple to calculate member forces. Conversely, statically indeterminant structures require that member stiffnesses be estimated. Estimating the member stiffnesses of a STM is often difficult because the true geometry of the struts are too difficult to be accurately determined.

STEP 5 – CALCULATE THE FORCES IN THE STRUTS AND TIES
The strut and tie forces can be calculated knowing the geometry of the developed STM and the forces acting on the D-region. It is desirable to use a computer program to calculate the forces because, often times, the geometry of the STM may need to be modified during the design process which will require the forces in the struts and ties to be recalculated.

STEP 6 – SELECT STEEL AREA FOR THE TIES
The required amount of reinforcement for each tie can easily be determined by dividing the force in the tie by the product of the yield stress of the steel and resistance factor specified by a design specification. The reinforcement chosen to satisfy the steel requirements must be placed so that the centroid of the reinforcement coincides with the centroid of the tie in the STM. If reinforcement chosen to satisfy the tie requirements cannot fit in the assumed location of the tie, the location of the tie in the STM needs to be modified, and the member forces need to be calculated again.

STEP 7 – CHECK STRESS LEVELS IN THE STRUTS AND NODES
The stress levels in all of the struts and nodes must be compared to the allowable stress limits given in design specifications. In order to determine the stress levels in the struts and nodes, the geometry of the struts and nodes must first be estimated. The geometry of the struts and nodes can be determined based on the dimensions of bearing pads and the details of reinforcement connected to the struts and nodes.

Accurately determining the geometry of internal struts and nodes not attached to bearing pads and reinforcement is more difficult than finding the geometry of struts and nodes directly in contact with the boundary of the D-region. In the case of internal nodes and struts, it may not be possible to precisely define the strut and node geometry. Brown et al. [9] explained that this uncertainty is acceptable because force redistribution can take place for internal struts and nodes.

When stresses in struts and nodes are found to be larger than permissible stresses, bearing areas, the reinforcement details, or the overall member geometry of the member can be modified in an effort to increase the overall geometry of the strut and/or node. When changing any or all of these items, the STM will likely need to be modified. If the STM is modified, the member forces need to be calculated again, the ties may need to be redesigned, and then, the stresses in the struts and nodes can be checked again. The concrete strength can be increased if modifying the geometry of the STM or the member itself is not possible.

STEP 8 – DETAIL REINFORCEMENT
Once all the steel chosen for the ties in the STM has been finalized, the anchorage of the reinforcement must be properly detailed in order for it to reach its yield stress prior to leaving nodal zones. In addition, appropriate crack control should be placed in areas that are expected to be subjected to cracking. Most design specifications specify a minimum amount of crack control that must be placed in a D-region that has been designed with a STM.

2.3.2 Example-Design of a RC Deep Beam with Openings using the Strut-and-Tie Method

This example Beam DSON3 in Group A shows how you can use the strut-and-tie method for designing a RC deep beam with openings.

2.3.2.1 Geometry and Loads

Figure 5.25 shows a simply supported rectangular beam 80mm wide and 400mm height carries one concentrated factored load of 140kN on a span of 800mm. The beam contains two rectangular openings 80×180mm, located in the shear-span of the beam. Given $f_c' =$, $f_y =$ 1 , f_{yv} = 244.5MPa (Ø6) , and f_{yh} = 260.2MPa (Ø8).

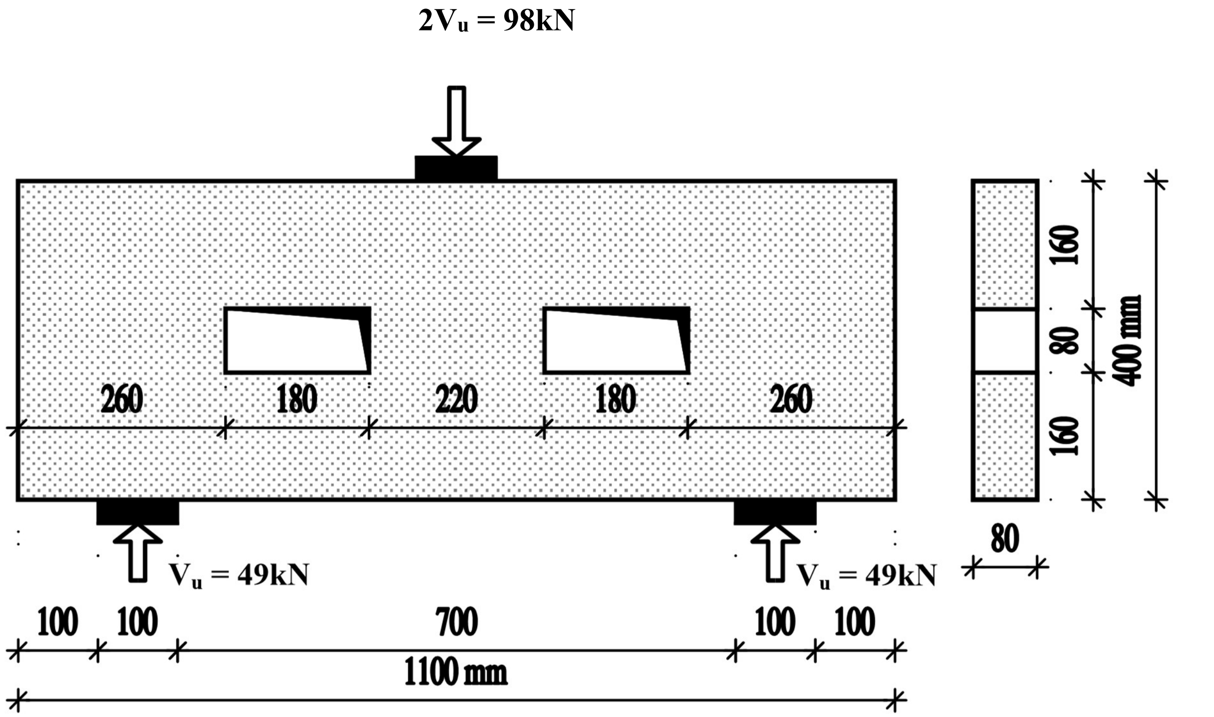


Figure 5.25 Beam geometry and loading.

2.3.2.2 Design Procedure

A deep beam is entirely a disturbed region because of loading and geometric discontinuities. The beam will be designed using the strut-and-tie method according to ACI 318-11. The step-by-step procedure is as follows:

Step 1: Check bearing capacity at loading and support locations.
Step 2: Establish the strut-and-tie model and determine the required model forces.
Step 3: Select the tie reinforcement.
Step 4: Check the capacity of the struts.
Step 5: Design the nodal zones and check the anchorages.
Step 6: Calculate the minimum reinforcement required for crack control.
Step 7: Arrange the reinforcement.

2.3.2.3 Design Calculations

Step 1: Check bearing capacity at loading and support locations

The area of bearing plate is 80×100mm. The bearing stresses are:

at point of loading: $$f_b = \frac{V_u}{A_c} = \frac{98 \times 10^3}{80 \times 100} = 12.25\text{MPa} \tag{5.39}$$

at supports: $$f_b = \frac{V_u}{A_c} = \frac{49 \times 10^3}{80 \times 100} = 6.125\text{MPa}$$

The nodal zone beneath the loading locations is a C-C -C Node. The effective compressive strength of this node (β_n=1.0) is limited to:

$$f_{ce}^{n} = 0.85 f_c' \beta_n = 0.85 \times 30.45 \times 1.0 = 25.88\text{MPa} \tag{5.40}$$

515

The nodal zone over the support location is a C-C-T Node. The effective compressive strength of this node (β_n= 0.8) is limited to:

$$f_{ce}^{n} = 0.85 f_c' \beta_n = 0.85 \times 30.45 \times 0.8 = 20.71\text{MPa} \tag{5.41}$$

According to ACI 318-11, the following equation shall be satisfied

$$f_{ce}^{n} \geq f_b \tag{5.42}$$

Where is a strength reduction factor, equals to 0.75 for all elements.

$0.75 \times 25.88 = 19.41\text{MPa}$ at point of loading

$0.75 \times 20.71 = 15.53\text{MPa}$ at supports

Since the applied bearing stresses (12.25MPa and 6.125MPa) are less than the limiting values at both the loading location and the supports, respectively, the provided area of contact (bearing plates) is adequate.

Step 2: Establish the strut-and-tie model and determine the required model forces

Based on the tension and compression stress trajectories (finite element analysis), the strut-and-tie model shown in Fig. 5.26 is proposed for Beam DSON3. Elements with compression stress trajectories will be replaced by compression elements (Struts) and elements with tension stress trajectories will be replaced by tension elements (Ties). Here, the compressive struts are shown in dotted lines while tension ties are shown in solid lines.

$2V_u$ = 98kN

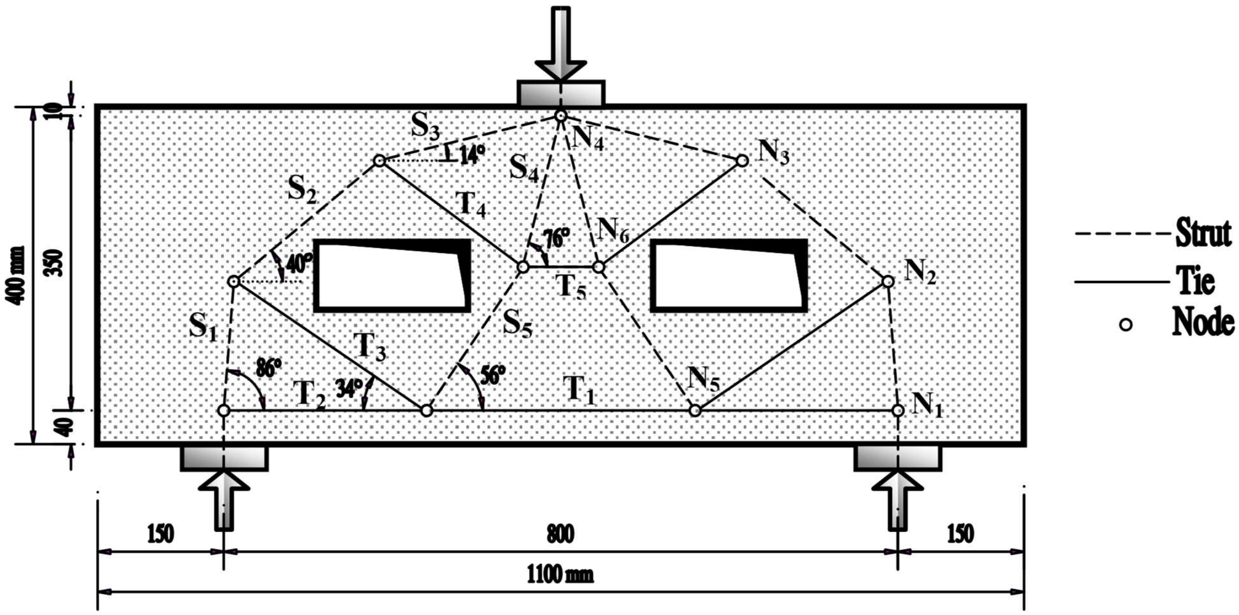


Figure 5.26 Details of the proposed simplified strut-and-tie model (using inclined ties) for Beam DSON3.

The reinforcement required for the inclined ties in Fig. 5.26 can be resolved into horizontal and vertical reinforcement, and the STM can thus, be adjusted as shown in Fig. 5.27. This model is better because it gives larger capacity. In this model, according to ACI 318-11, the smallest angle permitted between a strut and a tie in a D-region of 25 degrees is satisfied. The struts, ties, and nodes are labeled as in the Fig. 5.27. Referring to Chapter 3 (Sec. 3.3.2), the maximum nominal capacity of Beam DSON3 is 130kN. However, designing the beam according to the ACI 318-11, the ultimate load will be $F_u \leq F_n = 0.75 \times 130 \cong 98$kN.

$2V_u = 98$kN

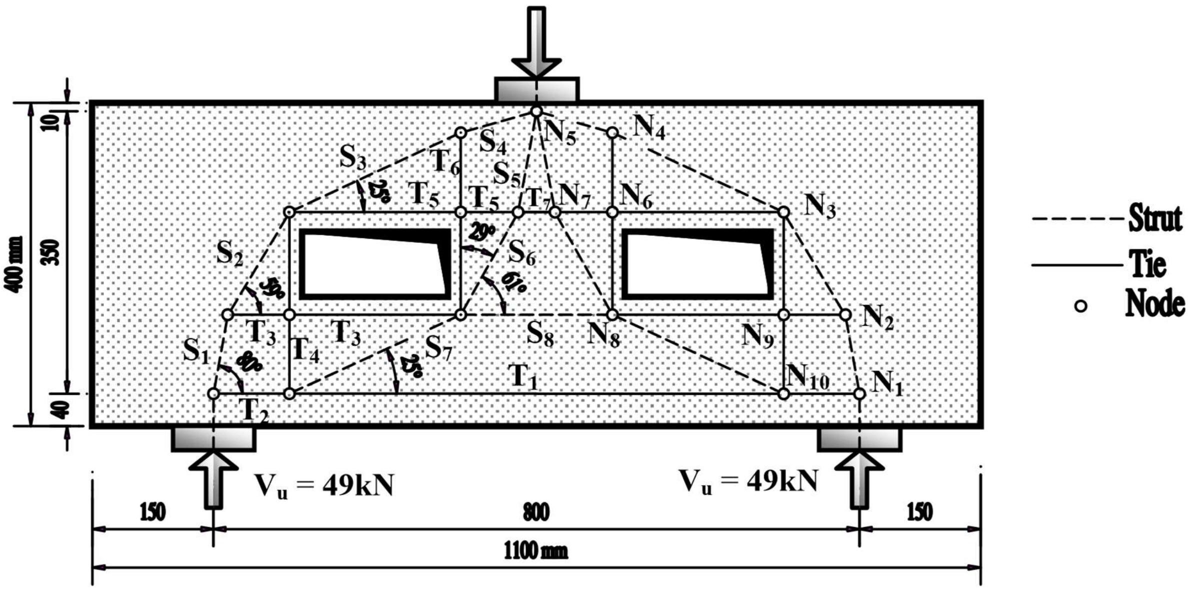


Figure 5.27 Alternative proposed simplified strut-and-tie model for Beam DSON3 (using vertical and horizontal ties).

The forces in all members are determined from statics and their magnitudes in kN are as in Table 5.3. The table also gives the inclination of each strut member (angle from the horizontal).

Table 5.3 STM forces for the strut-and-tie model in Fig. 5.27.

Model Label	Angle (Degree)	Force, kN	T or C	Model Label	Force, kN	T or C
S_1	80	49.97	C	T_1	54.59	T
S_2	59	56.73	C	T_2	9.80	T
S_3	25	69.21	C	T_3	18.78	T
S_4	16	66.42	C	T_4	20.36	T
S_5	80	28.33	C	T_5	34.43	T
S_6	61	32.41	C	T_6	7.64	T
S_7	25	49.20	C	T_7	22.40	T
S_8	0	9.70	C	--	--	--

T = Tension (Tie) and C = Compression (Strut)

Step 3: Select the Tie Reinforcement

The required amount of reinforcement for a tie will be determined as follows:

$$A_{st,required} = \frac{F_n}{f_y} = \frac{F_u}{\phi f_y} \tag{5.43}$$

Where F_u is the ultimate tensile force for the tie and   is a strength reduction factor and equals 0.75 for all the STM elements.

$$A_{st,required} = \frac{F_u}{\phi f_y} = \frac{F_u \times 10^3}{0.75 \times f_y} = 1333.33\frac{F_u}{f_y}$$

Table 5.4 Tie reinforcements.

Ties Label	Force, F_u (kN)	Yield Strength, f_y (MPa)	$A_{st,required}$ (mm^2)	Diameter (mm)	No. of Bars	Distribution
T_1	54.59	410	177.528	12	2 bars	2□12
T_2	9.80	410	31.870	12	2 bars	2□12
T_3	18.78	260.2	96.233	2	2 bars	2ϕ
T_4	20.36	244.5	111.029	3	2 stirrups	2ϕ6
T_5	34.43	260.2	176.428	8	4 bars	4ϕ
T_6	7.64	244.5	41.663	6	1 stirrups	1ϕ6
T_7	22.40	260.2	114.783	8	4 bars	ϕ

Step 4: Check the capacity of the struts

In order to check the capacities of the struts, the area of the struts must be first determined. The struts' areas were calculated by finding the product of the widths and depths of each strut. For the strut width w_s, refer to the dimensions of the struts in Fig. 5.28.

Knowing that f_c□ = 30.45MPa, the term $(f_{ce} = 0.85f_c'\beta)$ will be:

$f_{ce}^{sj} = 0.85f_c'\beta_{sj} = 0.85 \times 30.45 \times 1.00 = 25.88$ MPa, for Strut S_j (j = 1, 3, 4, 5, 7 and 8)

$f_{ce}^{sj} = 0.85f_c'\beta_{sj} = 0.85 \times 30.45 \times 0.60 = 15.53$ MPa, for Strut S_j (j = 2 and 5)

Maximum nominal strut capacity, $S_{jn,max} = f_{ce}^{sj} bw_j$ and ultimate strut capacity, $S_{ju,max} = S_{jn,max} = \phi f_{ce}^{sj} bw_j$. Table 5.5 summarizes the calculations performed for each strut.

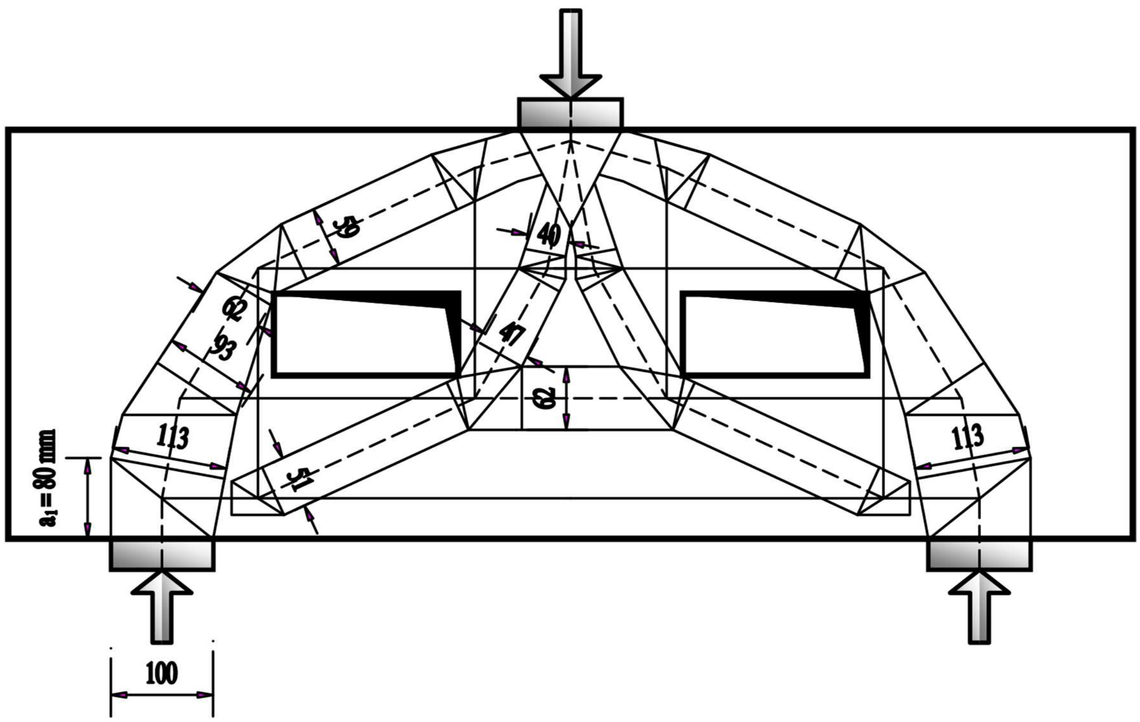


Figure 5.28 Visualization of struts' widths.

Table 5.5 Summary of concrete struts calculations.

Model Label	βs	f_{ce}^{sj}, MPa	Strut width	Ultimate strut capacity, $S^{ju}_{,max}$ (kN)	Actual Strut capacity (kN)	Okay
1	1.00	25.88	113.0	175.47	49.97	Yes
2	0.60	15.53	62.00	57.77	56.73	yes
3	1.00	25.88	59.00	91.62	69.21	yes
4	1.00	25.88	52.00	80.75	66.42	yes
5	1.00	25.88	36.00	55.90	28.33	yes
6	0.60	15.53	47.00	43.79	32.41	yes
7	1.00	25.88	51.00	79.19	49.20	yes
8	1.00	25.88	62.00	96.27	9.70	yes

Step 5: Design the Nodal Zones and Check the Anchorages

The capacity of a node is calculated by finding the product of the limiting compressive stress in the node region and the cross-sectional area of the member at the node interface. The maximum nominal node capacity is $f_{ce}^{ni} bw_i$ and $f_{ce}^{ni} = 0.85 f_c' \beta_{ni}$ and the ultimate node capacity is $f_{ce}^{ni} bw_i$. Table 5.6 summarizes the calculations performed for the critical nodes N_1, N_5 and N_8.

Table 5.6 Summary of critical nodes calculations.

Model Label	Type	βn	Surrounding Forces, kN	C/T	Available width, mm	f_{ce}^{ni}, MPa	Ultimate node capacity, kN	Actual capacity, kN	Okay
1	CCT	0.80	49.00	C	100.00	20.71	124.26	49.00	yes
		0.80	49.97	C	113.0	20.71	140.42	49.97	yes
		0.80	9.80	T	80.00	20.71	99.41	9.80	yes
5	CCC	1.00	98.00	C	100.0	25.88	155.28	98.00	yes
		1.00	66.42	C	52.00	25.88	80.75	66.42	yes
		1.00	28.33	C	36.00	25.88	55.90	28.33	yes
8	CCT	0.80	32.41	C	47.00	20.71	58.40	32.41	yes
		0.80	9.70	C	62.00	20.71	77.04	9.70	yes
		0.80	49.20	C	51.00	20.71	63.38	49.20	yes
		0.80	18.78	T	51.00	20.71	63.38	18.78	yes
		0.80	7.64	T	63.00	20.71	78.30	7.64	yes

According to the ACI 318-11, the 90° standard hook is used to anchor the ties T_1 and T_2. The required anchorage length is

$$l_{dh} = \lambda \frac{0.02 f_y d_b}{\sqrt{f_c'}} \tag{5.44}$$

$$l_{dh} = \frac{177.528}{226.195} \times \frac{0.02 \times 410 \times 12}{\sqrt{30.45}} = 14mm$$

Where $\lambda = \frac{required\ A_{st}}{provided\ A_{st}}$ represents the correction factor for excess of reinforcement. ACI 318-11 requires that this development length start at the point where the centroid of the reinforcement in a tie leaves the extended nodal zone and enters the span. As shown in Fig. 5.29, the available development length is 181mm. Because this is greater than 14mm, the anchorage length is adequate.

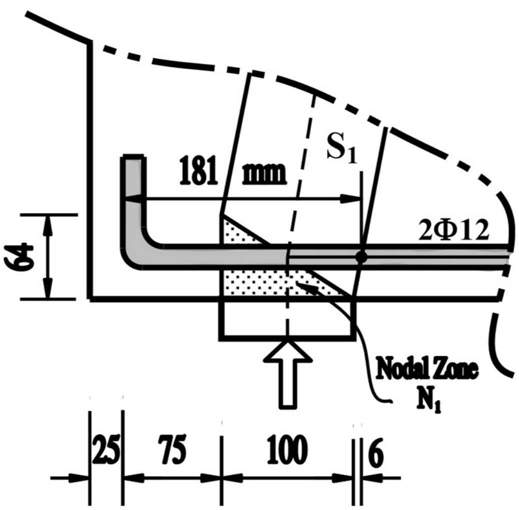


Figure 5.29 Nodal zone N_1.

Step 6: Calculate the Minimum Reinforcement Required for Crack Control

According to ACI 318-11, the provided vertical web reinforcement must be at least:

$$A_v \geq 0.0025bs \quad (5.45)$$

According to ACI318-11, the provided horizontal web reinforcement provided must be at least:

$$A_h \geq 0.0015bs_2 \quad (5.46)$$

Where s and s_2 is the vertical and horizontal spacing between web reinforcement, respectively and shall not exceed the smaller of d/5 and 300 mm.

For vertical web reinforcement, use Ø6-50mm on each side, $A_v/bs = (2 \times 28.27)/(80 \times 50)$ $= 0.014 > 0.0025$.

For horizontal web reinforcement use Ø8-50mm on each side, $A_h/bs_2 = (2 \times 50.3)/(80 \times 50)$ $= 0.025 > 0.0015$.

Step 7: Arrange the Reinforcement

The details of reinforcement are as shown in Fig. 5.30

Horizontal Web ϕ6 1ϕ6 Φ1

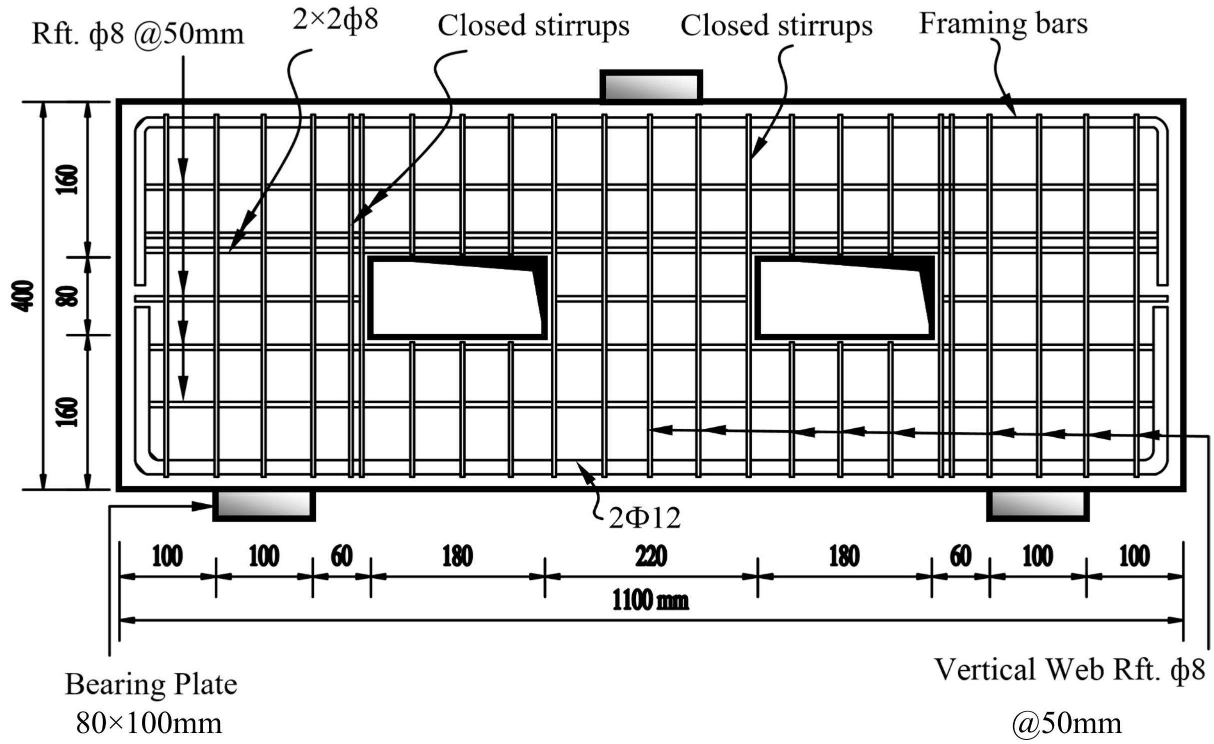


Figure 5.30 Final reinforcement detailing according to the strut-and-tie model.

2.4 DESIGN RECOMMENDATIONS

The following recommendations (which are not covered in the present study) are suggested for future researches:

- Based on the finite element output, future studies considering different types of top and bottom loadings (concentrated and distributed at various locations) to be done for simple shallow beams with openings.

- Continuous shallow beams with distributed loads, having openings with different shapes, sizes, and locations need more studies.

- The location of opening is a major factor influencing the strength of the beam. The effect of opening location on the strength and behavior of such beams have to be studied. This may yield the optimum location of openings in Beams.

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