Design of an iterative auto-tuning algorithm for a fuzzy PID controller

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Abstract. Since the first application of fuzzy logic in the field of control engineering, it has been extensively employed in controlling a wide range of applications. The human knowledge on controlling complex and non-linear processes can be incorporated into a controller in the form of linguistic terms. However, with the lack of analytical design study it is becoming more difficult to auto-tune controller parameters. Fuzzy logic controller has several parameters that can be adjusted, such as: membership functions, rule-base and scaling gains. Furthermore, it is not always easy to find the relation between the type of membership functions or rule-base and the controller performance. This study proposes a new systematic auto-tuning algorithm to fine tune fuzzy logic controller gains. A fuzzy PID controller is proposed and applied to several second order systems. The relationship between the closed-loop response and the controller parameters is analysed to devise an auto-tuning method. The results show that the proposed method is highly effective and produces zero overshoot with enhanced transient response. In addition, the robustness of the controller is investigated in the case of parameter changes and the results show a satisfactory performance.

1. Introduction

Since the first application of the fuzzy logic [1] in the field of control engineering field [2], an ever increasing employment of fuzzy logic controllers have been reported [3, 4]. They have been successfully applied in industrial processes and in some cases outperform conventional proportional-integral-derivative (PID) controllers [5, 6], in particular when the controlled system is complex or non-linear, as this is the case in many process control systems [7-9].

However the lack of a systematic method to design and tune these controllers may curtail their applications [10-12]. In general, the design of fuzzy logic controller involves three stages [12-14]. Firstly, the rule-base is constructed by translating the experience of a skilled human operator on controlling a plant into linguistic terms. Secondly, appropriate membership functions are selected. In final stage, the scaling gains of the controller are determined. To achieve better performance the rule-base, membership function parameters or the scaling gains are adjusted via trail-error-method or using optimization tool techniques such as: Genetic Algorithms (GA) [15], Ant Colony Optimization algorithm (ACO) [16], Shuffled Frog Leaping Algorithm (SFLA) [17] and Bees Algorithm (BA) [18].

The trial-and-error method is very simple and straightforward, but it is a tedious and a timeconsuming task [19], particularly when it is carried out on-line. Therefore, the technique is not always practical. In the second method, although these tools are powerful and their successes have been proved, they are computationally expensive [20]. Because they are population-based algorithms, a considerable number of solutions are generated; individuals of these generations are needed to be tested for their fitness functions. Additionally, some individuals cannot be tested in real-time and safety-critical applications; therefore, they are best suited for simulation based designs, where the plant transfer function is available.

Furthermore, there are other reasons that make the tuning process of fuzzy logic controllers more complex. It is difficult to find the relation between selecting membership function type or rule base, and the controller performance such as better rise time or less overshoot. In addition, unlike conventional controllers, fuzzy logic controllers have several parameters that can be adjusted [21], such as membership function shape, rules and scaling gains. Furthermore, there is no general rule of tuning these parameters. However, some techniques applied in tuning conventional controllers can still be utilised to some extent [12].

As in conventional PID controllers, there are various structures such as: fuzzy-proportional (FP), fuzzy-proportional-derivative (FPD), fuzzy-proportional-integral (FPI) or fuzzy-incremental (FInc) and fuzzy-proportional-integral-derivative (FPID) [13, 22-24]. Even for FPID controller, different structures have been proposed. A normal FPID with three inputs (error, change in error and integral error) has been proposed and implemented [24]. Although the controller has a simple structure, the construction of a three-dimensional rule-base becomes more difficult as the number of rules increases with the increasing number of inputs [13, 25]. Furthermore, constructing rules based on integral of error is rather difficult [13].

To overcome these limitations parallel structure (FPI+FPD) [25] and FPD+I [13] have been proposed. A Parallel structure which is a combination of FPI and FPD controllers has two inputs, resulting in a two-dimensional rule-base. Additionally, it has the basic properties of a general PID controller, but at the same time more computational time is required to compute the controller output as there are two fuzzy logic controllers in the structure. The FPD+I is constructed by combining a crisp integral action with FPD controller, hence the rule-base is two-dimensional and the controller has the merits of a general PID controller. Further configurations have been found in the literature such as: FPID with incremental output [26], rule coupled FPI+FPD [26], rule de-coupled FPID [26], FP+I+D [23], and FPI+D [23] controllers.

In this paper, an auto-tuning algorithm is designed to tune a fuzzy PID controller. The controller is applied to different second order systems. Initially, the controller gains are fixed and then automatically tuned to achieve the best possible performance. The results show that the proposed method is highly effective and produces zero overshoot with enhanced transient response. In addition, the robustness of the controller is investigated in the case of system parameter changes and the results show a satisfactory performance.

The remainder of this paper is organised as follows: section 2 presents an overview of the fuzzy logic controller structure and the fuzzy PID simulation design model. The auto-tuning algorithm is illustrated in section 3. Evaluation and simulation results are shown in section 4. Finally, some conclusions are drawn in section 5.

2. Controller design structure

In this section, detailed structure of the fuzzy logic controller and the simulation model are given.

2.1. Fuzzy PD+I controller structure

The Fuzzy PD+I controller reported in [13] is shown in figure 1. It was adopted as the controller in this paper; therefore its structure is illustrated in some details.



Figure 1. Fuzzy PD+I controller (FPD+I).

The controller consists of a normal FPD controller with added integral action; therefore it is known as FPD+I controller. The FPD controller action depends on the error (E) and the change of error (CE). The integral of error (IE) is then added to the output of this controller (cu) to form the FPD+I controller. The controller has the following scaling gains: gain of the error (GE), gain of the change of error (GCE), gain of the integral of error (GIE) and the output gain (GU). Signals are represented by lower case symbols before gains and upper case symbols after gains. These gains can be fixed or adjusted to achieve the best possible performance. The gains GE, GCE and GIE correspond to the proportional, derivative and integral gains in conventional PID controller respectively.

2.1.1. Fuzzification. To represent the values of inputs (E and CE) and output (cu), five symmetric triangle shape membership functions (except two trapezoids at the extreme ends for E and CE) with 50% of overlap were chosen [12, 13]. Although the choice of membership function shape and width is subjective, triangular shapes were chosen, because they are most popular and convenient [10, 12]. The interval of [-1, 1] was used for the universes of discourse of the input variables, while [-2, 2] was used for the output variable. The output universe of discourse is addition of the input universes; this is to achieve an approximate conventional PD controller which makes the controller tuning process easier [13].

The linguistic descriptions of the input and output membership functions are negative large (NL), negative small (NS), zero (ZE), positive small (PS) and positive large (PL). These are shown in figure 2 and figure 3 respectively.



Figure 2. Error and change of error membership functions.



Figure 3. Output membership functions.

2.1.2 Rule-base. The fuzzy PD rule-base is a mapping between the inputs and the output; it contains normal heuristic control rules of controlling a plant. A sample of the rules has the following form:

If error is PL and change of error is PL, then output is PL

The rule implies that if the error is positive large (measured output far away from the set point) and the change of error is positive large, then the control signal should be positive large to return back the output near the setpoint. As there are 5 linguistic variables for each input, 25 rules were created, table 1 shows the rules.

Controller Output (cu)		Change of error (CE)						
		NL	NS	ZE	PS	PL		
	NL	NL	NL	NS	NS	ZE		
Error (E)	NS	NL	NS	NS	ZE	PS		
	ZE	NS	NS	ZE	PS	PS		
	PS	NS	ZE	PS	PS	PL		
	PL	ZE	PS	PS	PL	PL		

Table 1. Fuzzy PD+I controller (FPD+I).

2.1.3 *Defuzzification*. The minimum (Min) operator was selected as an implication method, and the most popular and standard method of defuzzification process known as centre of gravity (CoG) was selected.

2.2. Fuzzy PD+I Controller Simulation Design Model

The fuzzy PD+I controller in a closed-loop feedback control system is shown in figure 4. The model was primarily comprised of the controller and a plant block. The error signal (e) was obtained from the difference between the setpoint (r) and the measured plant output (y). The change of error signal and the integral of error were produced by passing the signals through derivative and integral blocks respectively. A scope was used to show the closed-loop and the open-loop responses.

The closed-loop characteristics of the system were shown in the term of: maximum percentage overshoot (M_p) , rise time (t_r) and settling time (t_s) . A fixed-size (0.01 second) sampling interval was chosen.



Figure 4. Simulink model of the Fuzzy PD+I Controller.

3. The auto-tune algorithm

The auto-tune algorithm was comprised of two layers. The basic- layer was the Fuzzy PD+I controller, while an upper-layer was added to provide the capabilities of online identification,

adaptation and auto-tuning to the basic-layer controller by determining appropriate values of the controller gains based on the evaluation of the system performance. Additionally, it has the ability to monitor the performance of the system and to guarantee the stability.

The details of the algorithm can be summarised as follows. First, a closed-loop test on the system is performed by applying the fuzzy PD+I controller. The controller gains are set to their default values (one). The output is bounded and the overshoot is not allowed to exceed 100%, where the system becomes unstable. Secondly, if the response exhibits an overshoot with amplitude higher than 1%, the overshoot is measured and the values of GU and GIE are calculated as follows:

$$GU = M_p \tag{1}$$

$$GIE = 1 / (2 * M_p)$$
 (2)

Where M_p is the maximum percentage overshoot. This significantly reduces the overshoot. The gains are kept unchanged when the overshoot is less than 1%. Then, to improve the rise-time, the value of GCE is decreased. Finally, if the system performance is not satisfactory the value of GIE is increased. The last two steps are performed in an iterative base and the integrated square error (ISE) and the maximum percentage overshoot (M_p) were chosen to measure the performance of the controller.

The black diagram and the flowchart of the auto-tune algorithm are shown in figure 5 and figure 6 respectively.



Figure 5. The block diagram of the auto-tune algorithm.



Figure 6. The flowchart of the auto-tune algorithm.

4. Evaluation of the auto-tuning algorithm

4.1. Transfer Function Model

In order to evaluate the algorithm and to cover a wide range of systems, several standard second order systems with different characteristics were simulated.

Many real-time applications exhibit oscillation and overshoot in their step responses, these characteristics can be modelled using a second order system [27, 28]. Furthermore, this will help understand the response of higher order systems. Consider the standard second order transfer function [29, 30] in equation (3).

$$G_s = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
(3)

Where *K* is the gain, ζ (zeta) is the damping ratio and ω_n is the natural frequency. The system has different responses depending on the location of poles. The poles of equation (3) are the roots of the denominator and can be determined as:

$$p1, p2 = -\zeta \omega_{n\pm} \sqrt{(\zeta^2 - 1)} \omega_n \tag{4}$$

The value of ζ determines whether the poles are real or complex conjugate. From equation (3) suppose K = 1 and $\omega_n = 1$, depending on the value of ζ there are five distinct cases as following:

• If $\zeta \ge 1$, the poles are real:

- ζ = 1, critically damped system, denoted as case 1.
- ζ >1, overdamped system, denoted as case 2.
- If $0 \le \zeta < 1$, the poles are complex conjugate:
 - $\zeta = 0$, undamped (marginally stable), denoted as case 4.
 - ζ =0.5, underdamped system, denoted as case 3.
- If $\zeta < 0$, unstable system, denoted as case 5.

Different system transfer functions according to the value of ζ are shown in table 2 and the step responses of these systems are shown in figure 7.

Table 2. Second order transfer function	is by the value of ζ .
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Case	ζ	Transfer Function
1	1	$\frac{1}{S^2 + 2S + 1}$
2	1.5	$\frac{1}{S^2 + 3S + 1}$
3	0.5	$\frac{1}{S^2 + S + 1}$
4	0	$\frac{1}{S^2 + 1}$
5	-0.1	$\frac{1}{S^2 - 0.2S + 1}$





4.2. Auto-tune algorithm results

The controller with the auto-tune algorithm was applied to all the second order cases mentioned in the previous section. Due to the limited space of the paper only the step responses of the case 3 and case 5 which represent underdamped and unstable systems are shown in figure 8 - figure 13. The auto-tuned gains, open-loop and close-loop performance measures are shown in table 3.



Figure 10. Step response of case 3, iteration 11 - iteration 14.



Figure 12. Step response of case 5, iteration 6 - iteration 10.

Time

260

200

-0.2

16



Figure 13. Step response of case 5, iteration 11 - iteration 15.

Case	Auto-tuned gains		Open-loop performance measures			Closed-loop performance measures			
Cuse -	GCE	GIE	GU	M_p	t_r (second)	t_s (second)	M_p	t_r (second)	t_s (second)
1	0.3	0.129	16.28	0	3.358	5.834	0	0.661	1.194
2	0.2	0.102	19.69	0	5.858	10.660	0	0.497	0.818
3	0.3	0.078	12.72	16.3	1.638	8.076	0	0.598	1.014
4	0.3	0.089	22.34	100	1.020	N/A	0	0.520	0.922
5	0.3	0.067	29.76	N/A	0.9467	N/A	0	0.545	1.037

Table 3. Auto-tuned gains and performance measures.

The results obtained show that the controller was successful in controlling all the systems. The performance was substantially improved from the second iteration by eliminating the overshoot and then in an iterative manner the rise-time and settling-time were improved. This was approximately achieved in 15 iterations.

4.3. Step disturbance rejection test

After the completion of auto-tuning process all the systems were simulated for 100 seconds and tested with forcing a load of a step unit at time = 50 seconds. The responses are shown in figure 14 and figure 15. It is clear from the results that the auto-tuned gains were effective and the controller was able to overcome the disturbance.



Figure 14. Closed-loop step disturbance rejection test response of case 3.



Figure 15. Closed-loop step disturbance rejection test response of case 5.

4.4. Robustness test

To investigate the robustness of the algorithm in the case of system parameter variations, the transfer function in case 3 was considered here and tested. From the table 3, the auto-tuned gains were as follows: GE = 1, GCE = 0.3, GIE = 0.078 and GU = 12.72. The performance measures were as follows: $M_p = 0$, $t_r = 0.598$ and $t_s = 1$. 014. The normal parameters of case 3 were: K = 1, $\omega_n = 1$ and $\zeta = 0.5$. It was assumed that these parameters are changed by 20%, then the values become: $K = 1\pm 20\%$,

 $\omega_n = 1\pm 20\%$ and $\zeta = 0.5\pm 20\%$, therefore a total number of 27 sub-cases were tested and the performance measures were monitored and recorded as shown in table 3.

It can be seen from the performance measures in table 4 that in several cases the overshoot was increased up to 5.7%, also an increased settling time was noticed. However the closed-loop step response of all the sub-cases was stable with satisfactory transient response.

Sub-	Tran P	Transfer Function Parameters			Performance Measures			
case	K	ω_n	ζ	M_p	t_r (second)	t_s (second)		
1	1	1	0.5	0	0.598	1.014		
2	1	1	0.6	0	0.620	1.096		
3	1	1	0.4	0	0.576	1.787		
4	1	1.2	0.5	0	0.644	1.554		
5	1	1.2	0.6	0	0.670	1.608		
6	1	1.2	0.4	0	0.618	1.494		
7	1	0.8	0.5	4.710	0.593	2.456		
8	1	0.8	0.6	3.221	0.608	2.423		
9	1	0.8	0.4	5.042	0.577	2.485		
10	1.2	1	0.5	0	0.588	1.058		
11	1.2	1	0.6	0	0.608	1.133		
12	1.2	1	0.4	0	0.570	0.978		
13	1.2	1.2	0.5	0	0.622	1.341		
14	1.2	1.2	0.6	0	0.639	1.375		
15	1.2	1.2	0.4	0	0.604	1.305		
16	1.2	0.8	0.5	1.571	0.582	0.901		
17	1.2	0.8	0.6	0.926	0.601	0.936		
18	1.2	0.8	0.4	2.320	0.564	1.220		
19	0.8	1	0.5	0	0.621	3.778		
20	0.8	1	0.6	0	0.645	3.400		
21	0.8	1	0.4	0	0.597	4.276		
22	0.8	1.2	0.5	0	0.630	5.330		
23	0.8	1.2	0.6	0	0.657	4.868		
24	0.8	1.2	0.4	0	0.605	5.775		
25	0.8	0.8	0.5	4.366	0.653	3.155		
26	0.8	0.8	0.6	3.192	0.675	3.113		
27	0.8	0.8	0.4	5.719	0.631	3.192		

Table 4. Performance measures for change in parameters of case 3.

4.5. Case study

The transfer function shown in equation (5) represents two simple stages of mixing tanks, first order chemical reactors or heating systems connected in series [31]. A method has been proposed to design a PID controller in [31], where by relating the step response overshoot to the positions of zeros and poles of the transfer function, the parameters of the PID controller has been calculated. The controller has been applied to achieve zero overshoot response.

$$Gs = \frac{1}{(1+S)(1+10S)}$$
(5)

Three tests were conducted on the above system: the FPD+I controller with tuned gains using the auto-tune algorithm, the conventional PID controller as proposed in [31] and a conventional PID controller where Matlab auto-tuner tool was used to determine the controller gains . The parameters of the three controllers were as follows: for the FPD+I (GE = 1, GCE = 0.5, GIE = 0.017, and GU = 56.53), for the PID controller using the method in [31] (P = 7.2, I = 0.72 and D = 6.99. The original values were $K_C = 7.2$, $T_i = 10$ and $T_d = 0.972$ and they were converted to the values of P, I and D to be used within the setting of Matlab PID controller) and for the conventional PID controller using Matlab auto-tuner (P = 1.74, I = 0.20 and D = -4.58).

The closed-loop step responses of the three controllers along with the open-loop step response are shown in figure 16 and the performance measures of the controllers are shown in table 5.



Figure 16. Simulation results: (a) Open-loop. (b) Conventional PID (parameters found using Matlab auto tuner). (c) Conventional PID (parameters found using the method in [31].(d) FPD+I controller.

		Close-loop step response					
Performance Measures	Open-loop	Conventional PID (parameters found using Matlab auto- tuner)	Conventional PID (parameters found using the method in [31])	Fuzzy FPD+I (parameters found using the auto-tune algorithm)			
M_p	0	8.9	0	0			
t_r (second)	22.15	7.855	3.96	1.086			
t_s (second)	40.17	24.530	6.9	1.908			

Table 5. The performance measures of each controller.

It is evident from the results that the auto-tune algorithm was highly effective and response of the FPD+I controller has achieved zero overshoot with faster rise time and shorter settling time compared to other PID controllers.

5. Conclusions

An auto-tune algorithm for a fuzzy PID controller has been designed and applied to several second order systems. The results have been encouraging and indicate the validity of the technique where the performance of the system response progressively improves as the system is subjected to new step inputs. The results also showed that the algorithm was highly effective in achieving zero overshoot and produced a faster transient response. In addition, the robustness of the algorithm was investigated in the case of system parameter changes and the results showed a satisfactory performance.

A case study result showed that the auto-tuning algorithm outperformed the conventional PID controller in terms of achieving zero overshoot and faster transient response.

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